

STOCHASTIC MODELS OF PERSONAL
INCOME DISTRIBUTION IN SEGMENTED
LABOUR MARKETS

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TO MY LOVED ONES
MY PARENTS, YANNA, ALICE.

"TIME PRESENT AND TIME PAST ARE
BOTH PERHAPS IN TIME FUTURE AND
TIME FUTURE CONTAINED IN TIME PAST"

T.S.ELIOT.

I hereby declare that this thesis has been
composed by myself and the work is my own.

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Summary

The main object of this study is the growth and distribution of individual earnings in segmented labour markets. The thesis focuses on the micro-economic level and provides a stochastic framework for different wage structures or income promotion ladders.

The study critically discusses the Markovian property which characterizes the existing stochastic models of personal income distribution and, consequently, limits their applicability in dealing with different wage structures. The rigid formulation of these models does not provide an adequate insight into the distribution of earnings, since it does not allow for factors known to influence income promotion rates such as seniority within the income level and length-of-service.

The thesis also traces the various aspects of the segmented labour market hypothesis with particular reference to internal labour markets which exhibit stable employment relations and idiosyncratic job content at the firm level.

In the suggested stochastic models a change in earnings is identified with an event that occurs across the employee's length-of-service, and the growth of earnings is generated by a point process. Initially the distribution of earnings is derived within a firm subject to a particular wage structure or within a family of firms belonging to a single wage structure. On specific assumptions regarding the income promotion rates analytical solutions are obtained with particular attention given to the inequality of the distributions which is shown to depend upon seniority within the income level. At a later stage the distribution of earnings across different wage structures is also considered.

Since firms follow income promoting policies based on age, ability or length-of-service these factors are incorporated into the model. Finally a proposed model for the distribution of earnings of the self-employed and of workers in markets of the secondary sector takes into account income demotions.

The last part of the study provides a synthesis of the known consumption hypotheses focusing on the relation of individual consumption to earnings in internal labour markets.

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INTRODUCTION.

The models presented in this thesis are stochastic. Thus from a methodological point of view our work is based on the traditional application of stochastic processes to models generating size distributions of economic variables such as wealth, income and the size of the firm.

Apart from models specifically referred to in our survey of literature, chapter, one should mention the model of Wold and Whittle¹ who applied a birth-and-death process to the wealth variable, although their treatment of the increase of wealth is deterministic. Like most of these models in the field of economics Wold and Whittle assumed that wealth is a stock or a population of monetary units.

Steindl² treated the number of a firm's customers as the measure of its size, and applied a homogeneous birth-and-death process to the population of customers. In a similar model he applied a general random walk process to the firm's stock of capital, which increases with received claims and decreases with outlays on the cost of the business.

Adelman³ applied a Markov chain in order to describe the size distribution of firms and the concentration of the steel industry in the U.S.A. Newman and Wolfe⁴ examined the same problem for any industry. They divided the possible output range of an industry into a large number of size classes and applied a Markov chain. Within a quasi-stochastic framework they treated the transition matrix of the process as an increasing continuous function of price.

Finally, we should mention the more recent work of Shorrocks⁵ who applied a non-homogeneous birth-and-death process with "immigration" to a stock of monetary units

representing the wealth of an individual, and he derived the distribution of wealth within a homogeneous group of individuals having the same age.

Our claim for the originality of this thesis is three-fold. First we provide a mathematical framework within which earnings distributions in segmented labour markets, with different wage structures or income ladders, can be analyzed. The second novelty of this presentation lies in the application of point processes to the distribution of an economic variable such as income. These processes are basically non-Markovian and to our knowledge they have not been applied to economics. An original point in our exposition is the treatment of income promotion rates as akin to hazard functions, and the assumption - fairly general - that they are functions of seniority within each income level. We must note, however, that our models are not related to any man-power planning model. Therefore problems of vacancies or rationing of jobs are not considered significant and they are omitted, since income increases are not necessarily identified with job levels or promotions. Thirdly an interesting repercussion of our length-of-service model is the suggested relation between individual consumption and earnings in internal labour markets.

Although our models and the suggested consumption hypothesis can be subjected to empirical verification this task has not been undertaken due to lack of relevant data; this area still requires further research.

A shortcoming of the thesis is the assumption of constant exit rates of employees in our basic model. Perhaps a more realistic model would treat the exit rate

as a decreasing function of the length-of-service. Although we devoted substantial work to this area we realised that we could not obtain any explicit results, mainly due to integration problems. We based our results on the negative exponential form of the length-of-service distribution and on the theory of Laplace transforms.

A distribution that would not complicate the derivation of explicit results in our basic model and would be more realistic, is the mixed exponential with a probability density function of the form

$f(s) = p b m_1 e^{-b m_1 s} + q b m_2 e^{-b m_2 s}$, where b is the rate of entry into the firm per employee, m_1 and m_2 are two different rates of exit, and p, q are probabilities so that $p + q = 1$.

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CHAPTER 1: "FUNCTIONAL AND PERSONAL INCOMES DISTRIBUTIONS:

A DISTINCTION!"

The aim of this thesis is to provide a mathematical model for the growth and personal distribution of pretax money earnings (salaries and wages), which form the largest part of individual incomes; but as an introductory step it seems worth referring briefly to the theories of the functional incomes distribution and exposing the loose, and still questionable, connection that exists between these theories and topics strictly related to the personal incomes distribution approach.

The functional incomes distribution theory examines the incomes accrued to the factors of production; it is related to the division of national income into broad aggregates such as wages, rent, interest and profits. As far as the wage theory is concerned the functional incomes distribution concentrates upon the determination of the wage rate and upon the share of labour in the national income. The wage rate is defined as the price paid to labour per unit of time (usually per hour) for a particular kind of work.

On the other hand, the personal incomes distribution theory focuses upon the total income of an individual, the theory of earnings being a particular case of it.

A. The Classical Theories.

In the classical theories the functional incomes distribution is treated as being the outcome of social institutions and social relations. Prices and the rate of profit are simultaneously determined once the wage is known and, consequently, the determination of wages and of the surplus remains outside the prices scheme.

The concept of surplus or net product is of critical importance in the classical theory of distribution. Within the competitive capitalist framework the surplus is defined as the non-wage revenue, and it is determined by the methods of production and the wage, given the social output. The social output is composed of the requirements of production, accumulation and social consumption. Smith and Ricardo considered customs and habits as basic determinants of social consumption.

The classical economists drew an important distinction between "market prices" and "natural prices". The former are susceptible to influences of temporary or accidental causes deviating from natural prices which are considered relevant for the distribution problem. The fluctuations occur because of temporary monopoly advantages, changes in fashion, political events, etc.

Both the rate of profit and of prices are uniquely determined once the real wage is specified. The real wage is taken as given and according to Smith¹ it is determined by measuring the labour commanded.

Ricardo argued that the wage rate is determined by taking both the amount of labour employed and the total wage fund as fixed. Ricardo accepted the Malthusian doctrine of the natural price of labour, that is the equalization of

wages to the level necessary for the subsistence and perpetuation of the work-force. He, however, supported the thesis that the subsistence wage was socially determined by habits and customs.

The dynamic factor in Ricardo's analysis is profit; it results from the difference between wages and the product of labour on marginal land. The latter is determined in equilibrium by the fixed amount of labour, given technical knowledge and the fertility of land.

In the corn economy, the simplest version of Ricardian analysis, the rate of profit percent on capital employed is defined as the ratio of corn profit to the stock of corn^{*1}. In equilibrium there is a uniform rate of profit in all sectors of the economy, the corn rate of profit in agriculture regulating the money rate of profit in the industrial sectors.

Malthus³ raised the objection that wage goods do not consist of corn exclusively; if workers change their consumption patterns, the determination of the rate of profit entails the comparison of heterogeneous quantities such as total output, wages and total investment susceptible to price changes which in turn depend upon changes in the profit rate itself. This circularity problem^{*2} made the rate of profit seem indeterminate and rendered the underlying relations between profits and wages uncertain. Ricardo's quest of an invariant measure of value that would eliminate the interdependence between the rate of profit and relative prices did not end, even with the reluctant adoption of the labour theory of value, particularly labour embodied.

*1 See Blaug.²

*2 For a discussion of the circularity problem in the Ricardian theory of profit see Dobb⁴ and Mrs. Robinson.⁵

Finally, Ricardo pointed out that the limit to profits is set by the conditions of production and that there is an inevitable inverse relation between the share of wages and the profit rate. Malthus⁶ challenged this proposition of the inverse relation by arguing that a rise in wages could be compensated by a rise in prices, and hence would not necessarily be followed by a decline in the rate of profit.

Marx's theory of distribution is based upon the division of the total product between labour and capital. Marx⁷ suggested that the realization of profits can be explained in terms of the surplus value created in production by labour, since labour power contributes to the value of the product more than it receives. Therefore workers are exploited by capitalists, while wages are kept near some historically conditioned subsistence level by the industrial reserve army of labour created by the capitalist process.

According to Marx the distribution of income between capital and labour is influenced by the "class struggle", changes in technology, population growth, the organisation of workers and the expansion of capitalist production to new markets.

Both Ricardo and Marx were aware of the deviation of labour values from the competitive prices of production. The application of labour values as an invariant standard appears to be explained by Ricardo's effort to clarify certain analytical relations relevant to distribution theory and to establish them in the sphere of production. Marx argued that the labour theory of value not only clarified these distribution relations, but also offered an analysis to the process of generation and appropriation of surplus.

The non-equivalence of labour values and prices of production (the transformation problem) still remains one of the weaker points of the classical theories.

B. The Theories of Kalecki and Kaldor.

Between the classical theories and the much celebrated neoclassical theory stand the two theories of Kalecki and Kaldor.

Kalecki⁸ divides the economy into three sectors producing (a) investment goods, (b) consumption goods for capitalists and (c) wage goods. He makes the following general assumptions; first he considers a proportional rise of all wage rates in a closed economic system. As a result of the rise of wage rates the annual wage bill increases in a particular short period and the workers spend all their income immediately. Secondly the consumption and the investment decisions of capitalists are made before the short period considered so that they are not affected by the wage rise in this period. Since capitalists determine their investment and consumption on current experience they will not be inclined to reduce them just after the wages rise.

Kalecki examines two cases: on the assumption of perfect competition he argues that a rise in wage rates causes a proportional rise in prices at the given levels of respective outputs so that profits in sectors (a) and (b) rise in the same proportion as wages. In sector (c) profits will rise as well, since they are equal to the income obtained from the sales of wage goods to the workers of the sectors (a) and (b).

In the case of imperfect competition Kalecki assumes that each firm in an industry has the discretion to quote administered prices for its product on a cost and "mark-up" basis. The firm's direct costs u consist of the average costs of wages plus raw materials. The price set by the

firm depends on the degree of monopoly, that is on the relation of the ensuing price \underline{p} to the weighted average price of the product \hat{p} for the industry as a whole.

Therefore

$$\frac{p - u}{u} = f\left(\frac{\hat{p}}{p}\right)$$

where f is an increasing function; the lower p is in relation to \hat{p} , the higher the mark up will be fixed. The function f reflects semi-monopolistic influences and the higher the degree of imperfect competition, the higher is $f\left(\frac{\hat{p}}{p}\right)$ corresponding to a given relation $\frac{\hat{p}}{p}$.

The function f is, consequently, susceptible to Union pressure. The prices p may be different for different firms since firms usually experience different costs u or different functions f .

Kalecki argues that a general increase in wages, or an increase in all costs u with given functions f will make prices rise by the same amount. If the direct cost u_k increases only for a single firm, then the price set by that firm p_k will increase in a smaller proportion because the average price for the industry \hat{p} will not rise in the same proportion as u_k .

Under the assumption of excess capacities in an imperfect competitive structure, Kalecki concludes that in a closed system of this type a general increase in money wage rates will not change the distribution of national income, provided that the functions f particular to the various industries remain the same. There is, however, a possibility of redistribution if the Trade Unions manage to restrain the mark-up prices, but since the prices are not stable in the system a rise in wages is to a great extent shifted to the consumers.

Kaldor's model⁹ of functional incomes distribution is based upon the Keynesian equations

$$Y = C + I$$

$$I = S$$

$$\Delta S = s\Delta Y = \Delta I,$$

where total income Y is given (he assumes a state of full employment), s is the marginal propensity to save and the investment is also supposed to be given.

Kaldor assumes that there are two groups of income recipients, the wage-earners and the profit recipients, so that $Y = W + P$. Each group has a different marginal propensity to save, namely the workers' marginal propensity to save is s_w and the capitalists' one is s_p . The two partial marginal propensities to save combined give an average marginal propensity s that should satisfy the Keynesian equation $I = S$.

Kaldor considers simple proportional savings functions $S_s = s_w W$ and $S_p = s_p P$, where S_w and S_p are the aggregate savings out of wages and profits respectively, and the marginal propensities are equal to the average ones. Since $S = S_w + S_p$ we have

$$S = s_w W + s_p P \text{ or } \frac{S}{Y} = s_w \frac{W}{Y} + s_p \frac{P}{Y}.$$

But $\frac{S}{Y} = \frac{I}{Y}$, $W = Y - P$ and solving for $\frac{P}{Y}$ we obtain

$$\frac{P}{Y} = \frac{1}{s_p - s_w} \frac{I}{Y} - \frac{s_w}{s_p - s_w}. \quad \text{Accordingly the share of labour}$$

$$\text{is } \frac{W}{Y} = \frac{1}{s_p - s_w} (s_p - \frac{I}{Y}). \quad \text{Therefore the shares of profit and}$$

labour in income depend on the ratio of investment to output and on the two propensities to save, all known quantities.

The critical assumption in Kaldor's model is that the ratio of investment to output is independent of the changes in the

two savings propensities.*³ Assuming that investment decisions depend upon profit expectations the model is stable at full employment and only when $s_p > s_w$, the difference between the two propensities determining the degree of stability of the system.

*³ This particular point has been severely critized by Pen.¹⁰

C. The Neoclassical Theory of Distribution.

The neoclassical theory is based on the market mechanism and interprets relative prices as being formed at the equilibrium between the opposite and symmetric forces of demand and supply. Land, labour and capital are viewed as factors of production with their own markets in which the factor prices are determined by the operation of supply and demand forces as in the case of individual commodities. More particularly, the wage rate is determined by the supply and demand of labour. The demand curve of labour is derived from the demand curve of the final product it helped to produce.

The theoretical foundations of the neoclassical theory of distribution lie in the theory of marginal productivity. The idea of marginal productivity originated in the theories of von Thunen and Longfield^{*4} but this doctrine was completed more clearly developed by J. B. Clark^{*5} who identified it with a natural law.

The theory examines a hypothetical static state in which technical progress is absent and dynamic factors that could introduce an element of risk and uncertainty are not operative. Another basic assumption is the existence of perfect competition. In this system the only sources of income are wages, rent and interest, while entrepreneurial profit corresponds to reward for managerial labour; thus the origin and formation of profit is not explained by the theory.

^{*4} See Schumpeter.¹¹

^{*5} J.B.Clark¹² wrote: "The aim of my work is to show that the distribution of society's income is controlled by natural law, and that this law, if it operates unhindered, gives each agent of production the amount of the wealth that he has created."

Since the topic of this thesis is the distribution of earnings in segmented labour markets we will examine the theory of marginal productivity by focusing our attention upon the determination of the level of wages. According to the proponents of the theory the level of wages is determined by production optimization processes at the level of individual firms. The theory requires that the wage rate is equal to the marginal product of labour. The marginal product of any factor of production can only be defined by means of the production function and so far as labour is concerned it is the extra output produced by the additional employment of one extra worker; in mathematical terms the marginal product of labour is derived as the partial derivative of the firm's production function with respect to labour.

The neoclassical theory was extended from the micro-economic concepts to the field of aggregate values, and the shares of labour and capital in the national income are derived from the aggregate production function as the elasticities of the production function with respect to labour and capital. The neoclassical production function is of the form $Q = f(K, L)$, where Q is output, K is capital and L is labour; the share of profit is totally ignored. Two types of production function are known in the literature, the Cobb-Douglas one and the one known as Constant Elasticity of Substitution function. The Cobb-Douglas function¹³ is of the following mathematical form:

$Q = AL^a K^{1-a}$, where A is a constant and the exponents a , $1-a$ represent the shares of labour and capital respectively.

On the other hand the mathematical form of the CES function¹⁴ is

$$Q = \gamma \{ \delta K^{-\rho} + (1 - \delta) L^{-\rho} \}^{-\frac{1}{\rho}},$$

where the functional distribution of income is determined by the parameter δ .

In the Cobb-Douglas production function the elasticity of substitution between capital and labour is one, therefore the share of capital (or labour) is not influenced by capital accumulation, while the elasticity of substitution of the CES function is less than one, implying that capital accumulation increases the share of labour.

In the classical theory the primary problem was the identification of the principles governing the distribution of functional incomes, principles that were independent from those governing the production process; the functional income distribution process was treated as being the outcome of social institutions and social relations. The important feature of neoclassical analysis is that it renders the functional incomes distribution an essentially pricing process, since the remuneration of the factors of production is determined by the prices of the final products which in turn are explained by the conditions of exchange.

In the classical theory of distribution the value of the product is derived from its factor costs; prices and the rate of profit are simultaneously determined once the wage is known, while the determination of wages and of surplus remains outside the prices schema. On the other hand, the neoclassical theory of distribution traces the value of the factor from that of the product whose production it has contributed to, therefore the distribution and pricing problems are identically solved within the same process of price determination.

The marginal productivity theory has been criticized on both the question of finding the production optimum and the

problem of imputation.*⁶ The optimum combination of the factors of production is established through the comparison of successive increments in a certain production factor with the resulting increments in output, given that the level of technology and the prices of the factor and output are held constant. The optimum is defined as the maximum output for a given volume of production costs reached as an outcome of that combination of resources by means of which the value of the increase in output, with a marginal increment in any of the factors, equals the price of the latter. Therefore the basis of comparison is the elaboration of linear homogeneous production functions*⁷ of a microeconomic nature reflecting optimal quantitative dependences between output and cost, and also guaranteeing substitutability of the factors of production.

The practical consequences of this process, however, are limited for three reasons; first in many cases it is extremely difficult to calculate the necessary marginal values. Secondly the role of technical innovation is completely neglected, since we assume a given level of technology. Thirdly, the comparison of different combinations of factors entails the assumption of continuous substitutability of the production resources while in reality there are very few possibilities for the substitutability of factors, and these are largely determined by prevailing technical norms.

Another basic problem in the theory of imputation is the relationship between the marginal product of a factor and its remuneration. According to the theory the income of any

*⁶ For a critical discussion of the neoclassical theory see Alam.¹⁵

*⁷ See Schultz.¹⁶

factor of production is equal to its marginal product multiplied by the price of the final product in the commodity market. Consequently, total income equals the total marginal products of factors evaluated on the basis of commodity price; the value of commodities, being equal to the sum of income of the factors, becomes also equal to the total value of the factors' marginal products. The equalization of values of marginal products with the income of factors is realized through the joint action of the law of diminishing returns and the mechanism of perfect competitive markets. The entrepreneur in the perfect competitive market, seeking to obtain a certain amount of output at a minimum cost, arrives at an optimal combination of resources where the value of marginal products is equal to the corresponding income; this equality, however, does not hold when there are imperfections in the market.

D. Conclusions.

From this brief survey of the theories of functional distribution of incomes two conclusions can be derived. First the problem of the personal incomes distribution was not emphasized by the classical economists, while the models of Kaldor and Kalecki were solely concentrated on the factor shares.

Secondly the microeconomic part of the neoclassical theory provides an explicit answer to the problem of determining the size of the personal incomes but their distribution is not explained. Regarding the former problem, the difficulties mentioned earlier have not yet been overcome. The marginal productivity theory is not operative in an imperfect competitive market, while in many groups of wage or salary earners the concept of marginal product is rather vague and it cannot be easily determined, the services sector being an obvious example. But even if the marginal product of a worker can be estimated quantitatively there are other factors of institutional and sociological nature that may distort the picture, and jointly determine the wage structure. In addition, the Trade Unions from the supply side and the job evaluation schemes of large corporations from the demand side could seriously affect the distribution and determination of labour incomes.

It was generally assumed that there was some connection between the functional and personal distribution of incomes, but for the modern industrialized economies the relation appears to be ambiguous.

Bronfenbrenner¹⁷ points out that an apparent connection between functional and personal incomes distribution is incorporated in the following two propositions, we quote,

- "a) the larger the individual's income on the average, the smaller the proportion that is derived from wages and salaries,
- b) any measure that increases the labor share of the functional distribution will increase the equality of the personal one".

The author argues that the above propositions are half-truths. The major difficulty with the former is the existence of widows, orphans, pensioners and workers receiving supplementary incomes from rental of house property at the lower tail of the functional incomes distribution. The latter proposition is correct only if total income distributed does not fall at the same time; due to the fluctuations of profit, the labour share falls in prosperity and rises in depression, while the coefficient of concentration within the labour incomes should move upward in prosperity and downward in depression.

Bron fenbrenner, however, points out that there is some statistical evidence that shows that in fact the labour share is distributed more equally in prosperity since part time work and unemployment during the depression period increase the coefficient of concentration of labour incomes.

Our basic aim in this chapter has been to emphasize the fact that there might be some vague relation between functional shares and the coefficient of concentration of labour incomes, but the theories of functional incomes distribution do not explain the mechanism of the distribution of labour incomes.

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CHAPTER 2: "A SURVEY OF THE EXISTING LITERATURE"

A. A Brief History of the Personal Incomes Distribution.

We believe that a brief introduction to the pioneering work in the field of personal incomes distribution is worth being undertaken because, to say the least, it reveals the fallacies and short-comings that until recently prevailed. For many economists, nowadays, the quest of an overall size distribution of incomes is an obvious utopia, but for the two pioneers in the field, Pareto and Gibrat, it was the central point of research.

V.Pareto's law¹ was the empirical derivation of a curve from income tax data corresponding to various countries, such as Britain, Saxony, Peru, Florence etc. Pareto observed a certain regularity in the data and he thought that the following algebraic formula fitted the data well: $F(y) = Ay^{-a}$, where $F(y)$ is the number of incomes exceeding y , A is a constant related to the units of measurement and a is a parameter which controls all the known measures of inequality. The larger the value of the Pareto coefficient, a , the smaller the inequality and vice versa. Pareto believed that probability (chance) played little role in the generation of his universal law and that the numerical value of the coefficient a was nearly the same over the data to which he applied his law. The above two assertions, however, proved to be incorrect; we will see later on in this chapter that D.G.Champernowne derived the Pareto law from a Markov chain. In fact J.A.C.Brown² pointed out that by means of variations on the central limit theorem random processes can generate Pareto's law among other final distributions. So far as the second assertion is concerned empirical research has

found that the value of a varies with the degree of economic growth; in poor countries, where there is great inequality, the coefficient a has a small value, a figure of 1.5 according to J.Pen.³ In more developed countries with greater equality a higher value of a has been found. Pareto himself had also accepted the fact that a disaggregation of incomes by source gives rise to different curves related to different values of the coefficient. In chapter five we will establish that, according to certain assumptions in our basic model, the distribution of earnings in a firm can follow the Pareto distribution with the coefficient being governed by certain parameters of economic significance which can give rise to various curves of the Pareto form. Since, nowadays, it is universally accepted that the Pareto's formula fits well only the tail of the income distribution (the higher incomes) the validity of the Pareto coefficient as a measure of inequality in the whole income structure seems to be doubtful.

The only similarity that appears to exist between Pareto's law and R.Gibrat's log-normal distribution^{*1} is the empirical orientation of their work. Gibrat, unlike Pareto, focussed his attention upon the forces of chance, and by applying the powerful central limit theorem to his law of proportionate effect he derived the log-normal distribution of incomes. The law of proportionate effect, quoting from

^{*1} Actually Gibrat⁴ considered the more general case of $x = a \log (y - y_0) + b$ and showed that x is normally distributed. In the above expression a, y_0 and b are constants, y_0 corresponds to a minimum income, not necessarily the minimum subsistence value, and a is the only parameter which controls inequality.

J. Aitchison and J.A.C. Brown,⁵ can be defined as follows:

"A variate subject to a process of change is said to obey the law of proportionate effect if the change in the variate at any step of the process is a random proportion of the previous value of the variate".

Suppose that the following relationship links income at period t with income at period $t-1$:

$y_t = y_{t-1}(1 + \varepsilon_t)$. If the individual started with initial income y_0 , then the above relationship taken recursively becomes: $y_t = y_0(1 + \varepsilon_1)(1 + \varepsilon_2) \dots (1 + \varepsilon_t)$.

Taking the logarithm of both sides we obtain

$$\log y_t = \log y_0 + \sum_{i=1}^t \log (1 + \varepsilon_i). \text{ Provided that}$$

$\{\log (1 + \varepsilon_i)\}$ is a set of independent random variables, and for time t sufficiently large the random variables $\{\log y_i\}$, according to the central limit theorem, follow the normal distribution; in other words the income variate y is distributed as a two-parameter log-normal variate. The log-normal distribution fits fairly well the middle ranges of the income distribution, but it provides a poor fit for the tails. On the other hand, a particular deficiency of the log-normal distribution is that its variance increases as the process is continued (the longer the law of proportionate effect is in operation), while empirical evidence shows that the parameter $(\sigma^2 t)$ is constant over time.

M. Kalecki⁶ suggested that economic forces keep the standard deviation of the logarithm of income constant. This assumption implies a negative correlation between $\log y_{t-1}$ and $\log (1 + \varepsilon_t)$, and assuming that the regression of $\log (1 + \varepsilon_t)$ on $\log y_{t-1}$ is linear, we have $\log (1 + \varepsilon_t) = -a_t \log y_{t-1} + n_t$, where a_t is a constant and n_t is independent of $\log y_{t-1}$. Therefore the initial relationship becomes

$y_t = y_{t-1}^{1-a_t} e^{n_t}$ and under fairly general conditions

Kalecki has shown that the final distribution of the income variate y is approximately log-normal. The imposed operation of the negative correlation assumption is an artificial and rather arbitrary stabilizing influence which has been neither explained nor properly justified. J. Aitchison and J.A.C. Brown tried also to resolve the apparent inconsistency of the increasing variance by introducing a birth-and-death process of the income holders as a stabilizing factor. They argued that there is consistent evidence that in a number of professions (U.S.A. and U.K) the variance of the income distribution increases systematically with the age of the profession's members. More specifically the earnings of an individual through life are described by a stochastic process of the form $y_{t+1} = e^{\{f(t) + u\}} y_t$, where the function $f(t)$ describes the path of the median income through life and u is a normal variable with zero mean and independent of the individual's age t .

Although no actual explanation is given regarding the path of the median income, the fact that the individual's limited life-span constrains the variance of income seems to us a far more realistic assumption than the one suggested by Kalecki.

In our opinion Gibrat's absolute reliance on the central limit theorem is responsible for the confusion and scepticism which portray the economists' prevailing attitude towards stochastic models. It is generally believed that a stochastic model establishes the growth of personal incomes as the outcome of a large number of small additive random

effects which act independently from each other; it is true that human behaviour is unpredictable and this element of uncertainty, which is more obvious in the social sciences, can only be incorporated in the stochastic models. But, unlike Gibrat's model, there are certain economic sociological and institutional factors which systematically influence and, sometimes, determine the laws of probability.

At a later stage in this thesis^{*2} we will come back to this very important point that can be successfully summarized by J. Pen's expression "*The dice are loaded*".

*2

See chapter 4.

B. Stochastic Models of Personal Incomes Distribution

The observed similarity in the pattern of size distribution of personal incomes was the motivating force behind the application of stochastic models to the income distribution theory. The final size distributions were conceptualized as the steady state equilibria resulting after the stochastic process had been in operation for a sufficiently large time; and the final size distribution was seen to be the outcome of a great many random changes.

The processes applied were Markovian in nature, basically Markov chains, and they have been criticized on many grounds. The determination of the matrix of transition probabilities independent of the initial distribution does not give an insight into the latter, leaving it unexplained. The variance of income in these models increases with time contrary to existing evidence. We have to note, however, that stabilizing conditions were introduced to constrain the ever-increasing variance.

The major criticism raised against the stochastic models was that observed inequality is attributed to mere chance. More specifically this is related to the Markovian character of the models. By assuming that the transition probabilities are the same for all levels of income and independent of the former history of the process factors such as seniority within the income level, length-of-service, abilities or age are ignored. Finally the rigidity of the formulation excludes the possibility of the existence of different stochastic processes for various groups of individuals. Champernowne,⁷ however, considered this point by making allowance for the existence of homogeneous "colonies" of age or occupation groups with different transition matrices.

I Champernowne's Model (a Markov chain)⁸

Champernowne introduces a minimum income, say y_{\min} , below which incomes cannot possibly fall. Starting off from y_{\min} , an infinite number of successive income classes $j, j = 1, 2, \dots$ are marked-off by class limits that are equidistant on a logarithmic scale; if their width on such a scale is h , the j th income class refers to the income interval $y_{\min} e^{h(j-1)}, y_{\min} e^{hj}$. There is an enumerable infinity of these intervals. They can be made as small as we choose. Let us consider the movement across these income classes from one discrete time period to the next. An individual income recipient who at time t belongs to income class r may move to income class s at time $(t+1)$; the probability of this move is denoted as the transition probability $p(r,s,t)$. If $P(r,t)$ denotes the probability of an individual belonging to income class r at time t , we have $P(s,t+1) = \sum_r p(r,s,t)P(r,t)$..(1)

This equation brings out that the behaviour of the probability density $P(s,t)$ over time is determined by the transition probabilities $p(r,s,t)$. Champernowne makes a number of assumptions about these transition probabilities and some of these assumptions are justified only for large values of s and r (among the higher incomes). There are three main assumptions. First, the $p(r,s,t)$ are constants, independent of time. Secondly, they are equally independent of the income levels r and s , and determined fully by $u = s - r$, according to the "law of proportionate effect" the probability of a jump from one income class to another depends only on the width of the jump, but not on the position from which we jump off. Finally, income movement is restricted to the interval $(-k, 1)$ so that income may at

most move one class up or k classes down. It follows from these assumptions that we may write the transition probabilities $p(r,s,t)$ as $p(u)$, where $u = s - r$ and $p(u) = 0$ for $u > 1$ or $u < -k$, then we can replace (1) by

$$P(s,t+1) = \sum_{n=-k}^1 p(u) P(s-n,t) \quad \dots(2)$$

The process can be represented in terms of a vector $P(t)$, with elements $P(s,t)$, and of a transition matrix $A(t)$ with elements $p(r,s,t)$. In this notation (2) is written as $P(t+1) = A(t)P(t)$(3)

Since the transition probabilities are independent of time and apply to any income recipient who is in income class r at time t , regardless of his former history (i.e. the state of his income at $t-1$, $t-2$...), the process is known as a homogeneous Markov chain.^{*3} We may denote the transition matrix by A , deleting the time index, and it can be seen from (3) that the probability distribution $P(t)$ which is generated by the repeated application of the process to a given initial frequency distribution $P(0)$ is given by $P(t) = A^t P(0)$(4)

There are three restrictions related to the transition probabilities, $p(u) \geq 0$ for all u

$$\sum_{u=-k}^1 p(u) = 1$$

$$\text{and} \quad \sum_{u=-k}^1 u p(u) < 0,$$

the stability condition which means that the mathematical expectation of ranges shifted is always a shrinkage of income from whatever income one starts. This restriction

*3 The transition times related to the income changes, seen as events of the underlying point process, follow the geometric distribution which plays the role of the negative exponential one in discrete time models.

is needed in order to prevent the model from implying an ever-increasing variance.

Under certain conditions the repeated operation of the process ultimately will lead to a unique limiting distribution P^* which is independent of the initial distribution $P(0)$, that is $P^* = \lim_{t \rightarrow \infty} P(t) = \lim_{t \rightarrow \infty} A^t P(0)$ for all $P(0)$. This limiting distribution is the steady state distribution that satisfies $P^* = AP^*$, ..(5)

and it can be determined by solving this equation.

Champernowne states that the process defined for an infinite number of states by his model has the same property.

Through the operation of his process the income distribution will tend to a limiting distribution $P^*(s)$.

By analogy to (5) it follows from (2) that $P^*(s)$ must satisfy

$$P^*(s) = \sum_{n=-k}^1 p(u) P^*(s-u). \quad \text{..(6)}$$

Aitchison and Brown⁹ suggested that if the assumption of discrete income ranges is replaced by one of continuity, the resulting model is formally identical with the breakage process of Kolmogoroff. For, if the probability that a person with income in the interval $(X_t, X_t + dX_t)$ at time t will have income in the interval $(X_{t+1}, X_{t+1} + dX_{t+1})$ by time $t+1$ is denoted by $dG_t(X_{t+1}, X_t)$, the basic postulate of the law of proportionate effect asserts that $dG_t(X_{t+1}, X_t)$ depends only on t and on the ratio $\frac{X_{t+1}}{X_t}$. Therefore we can write:

$$dG_t(X_{t+1}, X_t) = dH_t \left(\frac{X_{t+1}}{X_t} \right), \quad \text{..(7)}$$

and the transition equation becomes

$$dF_{t+1}(X_{t+1}) = \int_0^{\infty} dH_t \left(\frac{X_{t+1}}{X_t} \right) dF_t(X_t), \quad \dots(8)$$

where $F_t(X_t)$ is the distribution function of the income variable X at time t . According to Aitchison and Brown the equilibrium distribution tends to log normality.

II Shorrocks' Modification to Champernowne's Model.¹⁰

Shorrocks suggests a simple modification of Champernowne's model by presenting a second-order Markov model where, we quote, "Individuals having been in income class i at time $t-1$ and now in class j at time t , have the same transition rates regardless of their income history prior to $t-1$." Let p_{ijk} be the probability of transition from income class j to k in the period $(t, t+1)$ given that income class i was occupied at time $t-1$. As in Champernowne's model p_{ijk} does not change over time and depends only on the size of the transitions $j-i$ and $k-j$ in the two periods $(t-1, t)$ and $(t, t+1)$. Thus $p_{ijk} = u_{k-j}^{j-1}$ and the first-order Markov case considered by Champernowne corresponds to the restriction that the u terms do not depend on the superscripts.

If we let $n_{ij}(t)$ be the expected number of individuals in class i at time $t-1$ and class j at time t , we will have the following equations:

$$n_{jk}(t+1) = \sum_i n_{ij} p_{ijk} \text{ for all } j, k \geq 0 \quad \dots(9)$$

$$\sum_k n_{jk}(t+1) = \sum_i n_{ij}(t) \sum_k p_{ijk} = \sum_i n_{ij}(t) \quad \dots(10)$$

$$\text{and } \sum_j \sum_k n_{jk}(t+1) = \sum_i \sum_j n_{ij}(t) = N \quad \dots(11)$$

where N corresponds to the total number of income holders.

Provided that $n_{jk}^* = \sum_i n_{ij}^* p_{ijk}$ for all $j, k \geq 0$, n_{jk}^*

will be an equilibrium distribution of the process.

Shorrocks obtains the solution for the equilibrium distribution by successive substitutions.

III Rutherford's Model.¹¹

Rutherford defines the logarithm of income as the "power" of income. Then he assumes that all income earners enter at the same age - in any given year there will be a stream of new entrants - and that these adult earners may receive both earned or unearned income. Rutherford also assumes a normal distribution of "income power" of the new entrants in the initial year. He mentions two reasons to justify his assumption,

- a) the distribution depends largely upon the distribution of parental income powers, education and social advantages, and
- b) the distribution depends upon natural abilities.

According to Rutherford these factors are normally distributed or very nearly so, although it is not certain whether they are so or not. Furthermore, he assumes that mortality is unrelated to income power at least in the modern welfare state and, by using the life tables of demographers, he estimates the proportion of those new entrants who will still be alive after any given period.

Let there be an entry of a_t individuals at time t , with income power distributed normally around mean M_t and with variance σ_t^2 . Then, assume that these new entrants are subjected to a random walk. The shock system in year $t+r$

will have a mean \bar{X}_{t+r} and a variance of U_{t+r}^2 and it will be normal in character. The distribution of income power at time T of the cohort which entered at time T-s will then be normal with mean

$$\begin{aligned}\mu_1 &= M_{T-s} + \sum_{r=0}^S \bar{X}_{T-r} \quad \text{and variance} \\ \mu_2 &= \sigma^2_{T-s} + \sum_{r=0}^S U_{T-r}^2\end{aligned} \quad \dots(12)$$

Rutherford assumes that the number of survivors at period $t+r$ of an entry of a_t at time t is given by $a_t e^{-kr} = N(r)$, where $\frac{1}{K}$ is the expectation of life at entry. So the mean and the variance at time T for the cohort entering at time T-s are:

$$\begin{aligned}\Sigma N(T-s) X &= a_{T-s} e^{-Ks} (M_{T-s} + \sum_{r=0}^S \bar{X}_{T-r}) \\ \Sigma N(T-s) (X - \bar{X})^2 &= a_{T-s} e^{-Ks} (\sigma^2_{T-s} + \sum_{r=0}^S U_{T-r}^2), \quad \dots(13)\end{aligned}$$

where $\bar{X} = M_{T-s} + \sum_{r=0}^S \frac{\bar{X}_{T-r}}{T-r}$.

The total distribution at time T will be the integral of the distributions of the cohorts entering at times T, T-1, T-2 ... back to the earliest cohort from which there are any survivors.

Rutherford makes the following simplifying equilibrium assumptions:

- a) There is a constant rate of new entrants, distributed with a constant mean and variance, that is $a_t = a$, $M_t = M$, $\sigma_t^2 = \sigma^2$, where a , M , σ^2 are constants.
- b) The shock system in all years has zero mean and a constant variance, $\bar{X}_t = 0$ and $U_t^2 = U^2(\text{constant})$.

Under these assumptions all cohorts, at all times, will

have the same mean, M . Consequently for the cohort entering at time $T-s$, we have at time T the following expressions for the mean and variance:

$$\sum N(T-s)X = M$$

$$\sum N(T-s)(X-M)^2 = a e^{-Ks} (\sigma^2 + sU^2). \quad \dots(14)$$

The moments^{*4} of the total distribution can be obtained by integrating each of the above relations over the range of $0 < s < \infty$. This gives, at time T , for any cohorts

$$\sum NX = M$$

$$\sum N(X-M)^2 = \frac{a}{K} (\sigma^2 + \frac{U^2}{K}), \quad \dots(15)$$

and the mean and variance of the total distribution are:

$$\mu_1 = M, \quad \mu_2 = \sigma^2 + \frac{U^2}{K}.$$

Finally, Rutherford determines the form of the distribution from the moments. This synthesis corresponds to the Cram-Charlier Type A series.

IV Mandelbrot's Stable Paretian Random Functions.¹²

The stochastic processes introduced by Mandelbrot as models for income distribution are based upon a family of stable non-Gaussian random functions (Pareto-Lévy law). Mandelbrot presents a theory of the stationary stochastic variation of income based upon the existence of certain limits for sums of random functions on which the effect of chance in time is multiplicative. In fact Parzen¹³ points out that these processes are filtered Poisson processes.

^{*4} The higher moments can be determined by following the same procedure.

A filtered Poisson process $\{X(t), -\infty < t < \infty\}$ is defined by $X(t) = \sum_{i=-\infty}^{\infty} \omega(t - \tau_i)$, where $\{\tau_i\}$ are the times of occurrence of events of Poisson type with intensity λ , and

$$\omega(s) = C|s|^{-b} \text{ if } s \geq 0$$

$$\omega(s) = -\omega(-s) \text{ if } s < 0, \text{ where } C, b \text{ are positive constants.}$$

Furthermore we have $b > \frac{1}{2}$.

The characteristic function of $X(t)$ is given by

$$\phi(u) = e^{-k|u|^a}, \text{ where } a = \frac{1}{b} \text{ and}$$

$$k = 2 a \lambda C^a \int_0^{\infty} y^{-a-1} (1 - \cos y) dy. \text{ When } 0 < a < 2$$

the above characteristic function is said to be a real stable characteristic function. For $a > 2$, $\phi(u)$ is not a characteristic function, and in the case $a = 2$, $\phi(u)$ is the characteristic function of a normal distribution.

V Simon's Model¹⁴

In Simon's model persons' incomes are regarded as populations which grow according to a certain population law, that is the number of births is in proportion to the size of the population. Simon's model was initially designed for the distribution of words in a text. It refers to a section of the text which is continuously changed by the addition of new words at one end and the dropping-out of words at the other. The same model was applied by Simon to the personal income distribution theory.

He considers the total annual income of all persons above some specified minimum being k dollars, the segment

of this sequence running from the m th to the $(m+k)$ th dollar is the income for the year beginning at time m . An income in this case is a population of i monetary units growing according to the population law. New income-earners enter the process with constant probability, while some income-earners drop out with their whole income (at the same average rate) so that the total segment of income remains unchanged. Simon pictures the stream of income as a sequence of dollars allocated probabilistically to the recipients and he bases his procedure on the following assumptions:

- 1) The probability that a dollar belonging to an income class i is dropped as m increases is $f(i)$.
- 2) To compensate for the number of dollars lost in this way, additional dollars are allocated to the various income classes, the probability that the additional dollar will be allocated to a person with an annual income of i dollars is proportional to $(i+C)f(i)$, where C is a positive small number which constrains the proportion of the total income stream going to persons of high income. $f(i)$ is initially regarded as the number of persons who possess i monetary units but later it is treated as a probability.
- 3) There is a constant probability \underline{a} that the additional dollar will be assigned to new income-earners, that is to persons very near the minimum income.

The process can be represented by the following equations in which m is the amount of dollars^{*5} which have been added to - and simultaneously subtracted from - the

*5

Alternatively it can be viewed as time.

total annual income of all persons:

$$f(i, m+1) = \frac{(1-a)k}{k+C} \{i-1+C\}f(i-1, m) - (i+C)f(i, m),$$

$$f(1, m+1) = \frac{ak+C}{k+C} - \frac{(1-a)k}{k+C} f(1, m). \quad \dots(16)$$

A steady-state equilibrium of the above equations is represented by

$$f^*(i) = \frac{(1-a)k}{k+C} \{i-1+C\}f^*(i-1) - (i+C)f^*(i)$$

$$f^*(1) = 1 - \frac{(1-a)k}{k+C} f^*(1). \quad \dots(17)$$

Simon shows that the solution for this birth-and-death process is the Yule distribution for large values of i .

VI The Galton-Markov Models.

The simple Markovian log-normal process sometimes called the Galton-Markov, motivated the introduction of various models into the personal income distribution theory. The log-normal model is usually written as

$$Y_{t+1} = Y_t + U_t, \quad \dots(18)$$

where Y is the logarithm of income, t is time and U are random variables independently and identically distributed with finite variance. The model is Markovian, thus Y_{t+1} depends on the present value Y_t but not on the previous values. On the additional assumptions that U are normal and independent of Y_{t-1} , the distribution of Y will tend towards normal after the process has been operating for time sufficiently large.

Kalecki, mentioned earlier, and Thatcher¹⁵ replaced the law of proportionate effect with the regression towards the mean by assuming that

$$Y_{t+1} - \bar{Y} = b(Y_t - \bar{Y}) + U_t - \bar{U}_t, \quad \dots(19)$$

where the value of b controls the acceptance of any of the two hypotheses.

Hart¹⁶ argues that, as far as the log-normal distribution of incomes is concerned, measures of static inequality based on the variance σ^2 are insufficient, and they should be supplemented by criteria related to dynamic properties of the income distribution, particularly on b , the logarithmic regression of a person's income at time t on that of for time $t-1$, and on the residual variance.

Thatcher¹⁷ incorporated the Galton-Markov process and Friedman's permanent income model into a single general model. More particularly, he suggests that observed earnings Y_t are given by

$$Y_t = y_t + \varepsilon_t, \quad \dots(20)$$

where y_t follows the Galton-Markov process and ε_t is a transient component. Equation (20) may be written as

$$Y_t = bY_{t-1} + U_t + \varepsilon_t, \quad \dots(21)$$

and if $\{\varepsilon_t\} = 0$ we obtain the Galton-Markov process, whereas if $b=1$ and $\{U_t\} = 0$ we obtain Friedman's model.

Finally Fase¹⁸ assumes a log-normal distribution for the initial income of persons at the age when they start their career, and relates age to the logarithm of income according to the following equation

$$X_T = X_{T-1} + \xi(\tau - T) + U_T, \quad \dots(22)$$

where U_T are random variables subject to the assumptions of the Galton-Markov, and $\xi(\tau - T)$ depends on age with $\xi > 0$ so that this log income component increases up to a certain age τ and then it decreases at the same rate.

In a similar model Creedy¹⁹ regards the proportionate

change in any individual's income as being the same as that of the geometric mean income of his age group, apart from a stochastic term which governs the extent of movements within the distribution. Symbolically we have

$$\frac{d \log y_{it}}{dt} = \frac{d \log m_t}{dt} + U_{it}, \quad \dots(23)$$

where y_{it} is the income of individual i at age t , measured in years from entry into the occupation or labour force, m_t is the geometric mean income in age group t , and U_{it} is the stochastic component.

C. Non-Stochastic Models.

In an attempt to give mathematical formulation to the neoclassical assumptions and to stress the importance of the supply side of the labour market the human capital models were introduced; one of the reasons for this development being the fact that they can easily comply with the familiar econometric relationships. We quote from Mincer²⁰ *"In stressing the role played by individual and family optimising decisions in human capital investments, important aspects of income determination are brought back within the main stream of economic theory and within the power of its analytical and econometric tools"*.

According to the human capital theory a simple earnings function for an individual is of the following form: $Y_t = Y_{t-1} + r_{t-1}C_{t-1}$, where C_{t-1} is the dollar amount of net investment in period $t-1$, Y_{t-1} are gross earnings including investment expenditures and r_{t-1} is the rate of return on this particular part of investment. The above expression taken recursively gives

$$Y_t = Y_0 + r \sum_{j=0}^{t-1} C_j, \text{ where } Y_0 \text{ is the initial earnings}$$

capacity with no investments made and r is assumed to be the same in each period.

The human capital approach treats the worker's choice as an important factor, thus failing to recognise that lack of mobility and the importance of promotional and on-the-job training opportunities often constrain the worker's future activities and earnings once his career origin has been determined. In many cases workers have a strong preference for permanent employment connections and for jobs yielding a steady income.

Lebergott²¹ examined the distribution of incomes of

males aged 25 - 64 and he found that the distribution is approximately normal. He attributed some skewness found in the distribution to the existence of institutional barriers, namely to the impact of credit rationing in truncating a normal distribution. Lebergott's point was that credit agencies deny credit to persons capable of losing large sums of money while affording credit to those who use it to acquire large incomes.

Lydall's model²² of the distribution of employment incomes within an enterprise was based on the assumption that the enterprise has the form of a pyramid with a specified number of persons in each grade. Each supervisor controls a fixed number of persons in the next lower grade and his income is directly related to the aggregate income of the persons whom he immediately supervises.

According to Roy²³ all workers tend to be rewarded in direct proportion to the output they produce. Each individual output depends on a great number of different uncorrelated factors which are considered to act together in a multiplicative way. By applying the Central Limit Theorem he concluded that the workers' output related to any commodity and, consequently, the workers' income is distributed log-normally.

Mayer²⁴ emphasized the correlation between a man's ability and his scale of operation measured, for example, in units produced. He defined ability as the probability of completing a given task successfully. His basic conclusion was that the value of one unit of a man's output follows the same distribution as his ability but on a different scale. Since the scale of operation is correlated with ability the distribution of the value of a man's output will have more

positive skewness or less negative skewness than the distribution of ability. Therefore, if we assume that ability is normally distributed the earnings will follow the log-normal distribution.

Stiglitz²⁵ attributed some of the earnings inequality to screening which is defined as the identification of qualities characterizing each individual. He argued that screening information has productivity returns but tends to increase inequality since an individual being labelled as more productive gets a higher wage, partly at the expense of others. Thus income distribution in economies characterized by imperfect information with respect to the qualities of individuals greatly differ from the one in economies with perfect information.

Finally Friedman²⁶ saw the satisfaction of men's tastes and preferences as a determinant of inequality in the personal income distribution. He assumed that alternatives open to an individual differ in the probability distribution of income they promise, and that individuals are free to choose among alternatives involving risk as if they knew the probability distribution of income attached to each alternative. The number of alternatives is limited by the additional assumption that individuals seek to maximize the expected value of their utility function which is an increasing function of income.

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CHAPTER 3: "THE INTERNAL LABOUR MARKETS"

According to Cain¹ labour economics has been a controversial field within economics since it covers such topics as income distribution, unions, unemployment and discrimination. The importance of labour economics is neither denied nor questioned by economists, and it is a well established fact that the neoclassical school played the dominant role in the field; a role that is based on the two familiar neoclassical assumptions, namely the profit maximizing behaviour of employers and the utility maximization of workers. In the 1960s, however, a new approach was developed challenging the conventional neoclassical theory, an approach that has been presented under various names, and consists of theories allied in purpose but different in ideological origin. The conceptual and empirical unit of the new approach is the segmented labour market and the names given to it, cited by Cain, are the following: radical, dual, tripartite, stratified, hierarchical, and job competition.

The most important feature of the segmented labour market theories is the existence of non-competing groups and particularly the dichotomy between primary and secondary sectors in the labour market. Our basic aim in this chapter will be to discuss the characteristics and functioning of the internal labour market, the basic structural unit of the primary sector, and also to put forward the theoretical foundations of a mathematical formulation for the growth and distribution of labour incomes in the above market.

A. The Description of the Internal Market.

Generally speaking, the internal labour market reflects the existing internal relations of modern bureaucratic corporations and it may be defined as an administrative unit within which the market functions of pricing, allocating and training labour are performed. These functions are determined by a set of administrative rules and procedures which are rigid in the face of the conventional economic variables, at least on a short-run basis. This set of institutional or administrative rules determining the internal structure of the market and its functions define, in particular, the ports of entry into the market, the relations between jobs and the privileges which accrue to workers within the market.

Internal markets form the major part of the primary sector - a well-known concept in the dual labour market literature - where, according to Doeringer and Piore,² there are a great number of good job opportunities in large corporations, stable employment relations, a high probability of promotion and good working conditions.

More recently Piore³ broadened the dual labour market hypothesis by dividing the primary sector into an upper tier and a lower one. The upper tier consists of professional and managerial jobs and, although there is still an internalized code of behaviour, there are no set of work rules and formal administrative procedures which characterize lower-tier employment relations. Educational requirements in the upper tier are an essential requisite for employment, jobs are highly remunerated and there are more promotion opportunities and greater status; the individual attributes considered to be important in the upper tier are general

skills such as initiative and creativity. From a sociological point of view, Piore links the upper tier to the middle class subculture and the lower tier to the working class one.

B. The Allocative Structure of the Internal Market.

A proper analysis of the allocative structure of the internal market has been made by the dual market theorists, the most prominent among the proponents of the segmented labour market theory. The analysis is based upon three factors:

- a) The degree of openness to the external labour market which depends upon the number of ports of entry and the restrictiveness of the criteria for entry. A closed internal market is one where all jobs are filled internally, while in an open one vacancies in all job classifications are filled directly from the external labour market. In the latter case hiring responds to external market conditions in the way hypothesized by the conventional economic theory but the allocation of work assignments, the determination of wages and the rationing of jobs are all governed by administrative rules that are not totally responsive to the market forces. We must note, however, that external allocations to higher grades rarely take place, employees normally entering the lowest grade and being promoted internally. Open internal markets mainly exist in the secondary sector, and especially in the case of self-employed persons.^{*1} Nevertheless, external allocations, when they occur, tend to be restricted to firms or institutions within a certain industry; they seldom take place across them. Therefore the broader and more general the organizational structure being considered the smaller the probability

^{*1} See table at the end of the Chapter.

of an external allocation.

Bosanquet and Doeringer⁴ in discussing the existence of dual labour market in Great Britain divided internal labour markets into structured and open ones on the basis of enterprise-specific training, the term "structured" being equivalent to the term "closed". Firms in open internal markets allocate most jobs externally and provide few opportunities for on-the-job training and promotion. On the other hand, enterprises in structured internal markets hire workers into a limited number of entry jobs, and then promotions take place internally with emphasis given to specific training. Whenever firms invest heavily in specific training they rely upon internal promotion ladders and upon the minimization of labour turnover. If the job structure within a firm requires few specific skills, then most of the jobs will be filled from the external labour market, and the employer is generally indifferent to turnover. The authors conclude that structured internal markets are found mainly in the primary sector, while the open ones exist in the secondary sector of the market. The presence of enterprise-specific training encourages market duality not only on the demand side of the market but also on the supply one, since workers in the primary sector have access to opportunities for specific on-the-job training that reward stable employment relations. Habits and traits developed by workers in both sectors tend to reinforce the polarization (the segmentation) of the labour market.

- b) The geographical and occupational scope-size of the internal market define mobility clusters, that is group-



ings of jobs within which patterns of mobility are explicit. Jobs in each mobility cluster share one or more of the following elements: related skills and work experience, similar level of job content, a common departmental organization and a single focus or work. Mobility clusters have a vertical and horizontal dimension; the former is defined by the range of skill content of the jobs within the cluster, while the latter measures the number, degree of specialization and, perhaps, diversity of the jobs at any level of skill.

Birnbaum⁵ presented a similar classification by arranging jobs in skill families on the basis of skill relationships. In each skill family job tasks differ according to the level of skill each involves (position of an imaginary learning curve), and also jobs differ in the range of skill families they encompass.

- c) There is a set of rules determining the priorities in which workers will be distributed among the jobs within a mobility cluster. These rules are functions of ability, seniority and frequency of work. Needless to say that there is a unity and coherence in the rules governing the internal allocation of labour so that a change in any one dimension will require adjustment along other dimensions as well.

C. The Generation and the Origins of Internal Markets.

The prevalent view among segmented labour market economists is that the intensification of hierarchial control and the internal specialisation of labour brought about the creation of job ladders, particular entry-level jobs and promotion policies. A more detailed analysis of the emergence of internal markets, however, requires an outline of the three main hypotheses presented so far regarding the generation of these markets.^{*2}

First the dual market theorists consider technology and custom as the two main factors that initiated internal labour markets. There are specific skills uniquely attached to certain job classifications in a firm, and due to this skill specialization a very small number of individuals learn a particular skill at a given time in the process of production. Therefore, on-the-job training is far more essential than general training acquired by education, and since labour is considered to be a quasi-fixed factor of production, employers face increasing turnover costs. The other crucial factor involved in the generation of internal markets is custom and practice which accounts for the rigidity of the prevailing administrative rules and guarantees stability for both employer and the workforce. Customary practice ensures the transition from the competitive market to the internal one provided that costs can be reduced; the costs being considered are usually the value of an internal market to the labour force (the trade-off between present and future income security), the turnover costs and the technical efficiencies of the internal market in the recruitment, screening and training of labour.

^{*2} None of these hypotheses has been adequately substantiated.

Wachter ⁶ takes a similar approach within the framework of the neoclassical theory by arguing that efficiency and productivity play the only important role in the creation of internal markets. As in the dual market theory, he bases his argument on the existence of idiosyncratic jobs and the technology of on-the-job training, but he stresses the efficiency orientation of firms in internal markets. A modern bureaucratic corporation has to minimize bargaining and turnover costs, and also to ensure that investments of idiosyncratic types, a potential source of bilateral bargaining for jobs, are undertaken without the risk of exploitation by either the unions or other firms. Wachter lists three functions of the promotion ladder in an efficient internal market - namely, rewarding meritorious performance and reducing turnover, the acquisition by workers of specific training for higher-level jobs in the firm and the provision of a screening mechanism within the firm. So far as the last function is concerned the author points out that firms possess imperfect information with regard to screening and, although schooling and educational credentials play some part in providing useful information, in a technology that requires specific and on-the-job training the promotion ladder may be the most efficient apparatus for collecting and analyzing data on individual performance.*³

*³

Reder ⁷ claims that a substantial part of human capital investment takes the form of employer knowledge of employee capabilities, behavioural traits or suitability for promotion and of specific training at any stage of the promotion ladder. Therefore the greater costs of specific training, hiring and screening induce employers to favour promotion from within rather than hiring outsiders.

Finally the radical school explains the appearance of internal markets within a more general framework by attributing it to the institutional arrangements governing production. Edwards⁸ argues that there are two systems of control:

- a) simple hierarchy where the control is openly, arbitrarily and personally exercised by the capitalist, and
- b) Bureaucratic control which is a more sophisticated, impersonal and institutionalized means of exercising power.

These two systems of control are related to the social relations of production in the firm.

In the early era of monopoly capitalism big corporations tried to gain control over the risky elements of their environment, particularly the labour force, by internalizing them; thus, as large firms moved from simple hierarchy to bureaucratic control, they initiated the creation of internal markets. The imposition of bureaucratic control altered the power relations of hierarchical authority by rendering them invisible and embedding them in the firm's structure. Jobs became thoroughly differentiated by their organizational and technical aspects, and the supervisor is seen as a monitor and evaluator of the employee's performance. According to the radical theorists, the basic outcome of this process is that the workforce became less hostile to an impersonal and massive organization, while behaviour characteristics associated with bureaucratic control - such as propensity to follow the rules, habits of predictability, dependability and loyalty to the firm - became essential.

Bureaucratic control also encourages the development of employment stability by creating career ladders and instituting rewards or promotions for tenure and seniority within the firm. Rewarding the workers who demonstrated stability, both in their everyday behaviour at the job, and in the length of time they stayed at a job, is seen by the radical theorists as the main feature of the operation of internal labour markets.

In retrospect dual theory sees technology as the main determinant of the development of internal markets which, under certain constraints, are efficient systems of work organization. On the other hand, radical theory considers internal markets as systems with which capitalists maximize their control over the labour force, since homogeneity (non-segmentation) of the labour force would strengthen workers' resistance to ruling class domination of the production process. In both theories Trade Unions play a marginal role; this particular point has been raised by Rubery⁹ who argues that workers can establish and maintain a bargaining position through the development of a structured labour force, and that in both dual and radical theories the development of the structure has been viewed from only one perspective, through the motivations and actions of capitalists, the role of worker organization having been completely neglected in the process.

Rubery relies upon Braverman's thesis¹⁰ that the period of monopoly capitalism has been characterized by a progressive reduction in skills for the mass of workers; more particularly the greater amount of scientific knowledge embodied in the labour processes has brought about a polarization of skills in the sense that the mass of workers have lost craft

and traditional abilities, and they have been alienated or lost control over the sophisticated labour processes.

Braverman's attack is two-fold, first he criticizes the statistical classification of occupations in the U.S. with special reference to the category of semi-skilled workers who, due to some connection with machinery, have been arbitrarily called semi-skilled without possessing any real training or ability and have increasingly grown in numbers. Therefore there is a continuous redistribution of employment to the occupations where productivity increases have been lowest, and a progressive reduction in divisions within the mass of the labour force, based on a levelling down of the skilled workers.

His second criticism is addressed to the lengthening of the average educational period; contrary to the human capital approach where an increase in the standards of education and in the general productivity level of workers causes a levelling-up of skills, Braverman argues that the functions of education are primarily to create obedient and disciplined citizens who can conform, and adjust to the urban environment, to provide socialization to the city life and to decrease the recognised level of unemployment. He concludes that education has been losing its connection with the job or occupational requirements, and he implicitly questions the importance of job-specific skills in the monopoly capitalism era, thus undermining the dual market theory.

Rubery, on the other hand, suggests that the internalization of the labour market was brought about by the workers' success in regaining some of the control lost through the destruction of the craft system rather than by

capitalists' efforts to increase their control over the labour force. Workers, in order to safeguard themselves from the threat of substitution - due to the introduction of new technology - and competition, try to differentiate themselves from potential competitors, thus depending upon various systems of labour segmentation. The development of monopoly capitalism continually disrupts industrial organization and displaces labour as skills become redundant. The movement of capitalism from old into new areas destroys existing jobs and skills, the serious wide-effect being the reduction of the workers' bargaining power due to the competition for jobs.

D. The Dynamic Analysis and the Evolution of the Structure
of Internal Markets.

It seems to us that internal labour markets, regardless of their origin, are uniquely related to the advanced capitalist era.^{*4} Braverman's argument that the process of development deskills the working-force and prevents the creation of on-the-job specific skills suffers from three serious shortcomings. First, as Rubery pointed out, he completely ignores the resistance of the labour force within a dynamic framework of analysis and, in particular, the Trade Unions role in the determination of the social relations of production. Secondly, by being preoccupied with craft expertise, he ignores the fact that specific forms of skill or expertise may be tolerated by capitalists, provided that these skills do not present major problems to the expansion (or accumulation) of capital. During periods of technical innovation technological change creates new skills or increases the productivity of existing ones, thus rendering these skills essential to the production process; the computer and electronics industry is a well-known example.

*4 Some empirical evidence on the existence of internal markets has been collected by Doeringer and Piore; the authors do not present their evidence, but they caution the reader against the problems of fragmentation, sampling and statistical design related to the data.

Bosanquet and Doeringer point out that there is market duality in Great Britain, but due to apprenticeship tradition in the British industry, and the strength of craft unionism internal markets are not as structured as in the U.S.A. Their conclusions are mainly based on their own survey and on the work by D.I. Mackay et al., Labour Markets under Different Employment Conditions, Glasgow University, Social and Economic Studies, No.22 London 1971.

Finally, Alexander¹¹ provides some empirical evidence on the existence of manorial (closed), guild and unstructured internal markets, employee mobility being his classification criterion. He derives his data from the Social Security one percent work history file in the U.S.A.

Thirdly, Braverman disregards the very nature of internal markets with their permanent employment relations and their committed labour force. Hierarchical job ladders and permanent employment connections not only foster on-the-job training programmes, but they also enable the workers to familiarize themselves with the existing production process in the firm and, consequently, to disrupt it by collective action.

The evolution of the internal market structure is determined by both the demand and the supply side of the market, the interaction between these two forces being too complicated to be analyzed in detail. From a static point of view the basic determinants of the structure are the technology of production and the work methods within a firm set by entrepreneurs and Trade Unions. From a dynamic point of view, on the demand side, technological change and qualitative and quantitative shifts in demand for the product of the firm can seriously alter the structure of the internal market by affecting the job content, the proportion of specialized jobs and the employment opportunities within the plant or the firm. On the supply side, the pertinent factors are custom, tradition, Trade Union pressure and the availability of various types and qualities of labour in the external labour market.

In many cases, the supply forces tend to restrain the efficient development of the market structure; the rigidity of custom and practise and the workers' interest in the market as a means of enhancing job security and advancement act as constraints upon the employers' initiative to choose the structure that minimizes costs. The latter constraint can take any of the following forms:

- a) Control of the experienced workforce over the supply of skilled labour.
 - b) Worker expectations regarding the rules affecting the relationship between present and future incomes within the internal market.
 - c) Restrictions upon entry, that is a rigid set of entry rules that provides job security and insulates workers from external competition.
 - d) Dependence of promotion and allocative rules on seniority which also insulates workers from external competitors.
 - e) Rationing of jobs within the market in order to prevent a change in the rules that could bring about conflict between various groups of workers.
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E. The Wage Structure of the Internal Market.

Doeringer and Piore¹² recognize three dimensions of the internal wage structure:

- a) The plant wage level relative to other plants (or firms) in the community, or the weighted average of plant wage rates.
- b) The vertical differentiation of the wage structure which refers to differences in wage rates among individuals holding different jobs (skill grades).
- c) The horizontal differentiation which refers to differences in wage rates among individuals holding the same job.

According to the authors, the formal instruments of wage administration are job evaluation schemes, community wage surveys and engineered production standards. Job evaluation is a procedure for ranking jobs based on a consistent set of weighted job characteristics and worker traits. Jobs are grouped in labour grades and a certain wage rate is assigned to each grade, thus forming a grade hierarchy. One main purpose of job evaluation is to make the wage rates paid to employees wholly dependent upon the jobs they hold, so that personal favouritism can be either eliminated or, at least, minimized.

By a community wage survey the firm appraises the appropriateness of its wage structure with respect to its major competitors for labour, identifies a target wage structure, and tries to adjust its own structure to the target one.

Finally, the engineered production standards are related to horizontal differentiation; by merit rating and incentive systems employers evaluate and reward good

individual performance at a single job grade. Among these instruments the most important one is the job evaluation scheme.

The above described instruments of wage determination in the internal market reflect stable market conditions and usually adjust to any market change. One would expect, however, a divergence or conflict between labour market evaluation of jobs and internal job evaluation but, as Reder¹³ pointed out, these conflicts are not very frequent because market considerations enter indirectly the internal job evaluation process by:

- a) The existing similarity of qualities that add to the value of a job in both the labour market and the job evaluation scheme.
- b) The prevailing tendency to choose the weights attached to certain job factors in the internal evaluation process so as to yield comparatively high point values for those types of workers to whom the firm must pay relatively high wages, as a condition of securing and retaining their services.

Employers foster a formalized wage setting in order to alleviate Trade Union pressure and to advance intrafirm efficiency. The first point is related to the firm's efforts to justify their wage policies to Trade Union representatives and to equity-conscious workers for the purpose of reducing turnover, developing company loyalty and improving employee morale.

So far as intrafirm efficiency is concerned, the internal wage structure, apart from the entry job the wage rate of which is determined by competitive market forces, attaches wage increases to promotions in order to make them

desirable, thus encouraging employees to absorb more training and establishing authority relations within the firm.

A more precise and broader classification of internal wage rates was presented by Dunlop¹⁴ who introduced the concepts of job cluster and wage contour, the central points where the wage-making forces are concentrated. A job cluster is defined as a stable group of job classifications or work assignments within a firm linked together by technology, social custom and the administrative organization of the production process. The jobs within the cluster have common wage-making characteristics and the wage rates are closely related.

Dunlop argues that any internal wage structure is divided into groups of job clusters. A wage contour is defined as a stable group of wage-determining units (firms or plants) which are linked together by similarity of product markets, resort to similar sources for a labour force, common labour market organization and common wage-making characteristics. Therefore a wage contour has three dimensions:

- a) Particular job clusters.
- b) An industrial sector, and
- c) Geographical location.

We must note, however, that a wage contour should not be identified with an industry, it is rather confined to certain ranges of skill, occupations, or job clusters of the constituent firms.

In a wage contour the wage rates are mainly determined by the key firm or firms, a key firm being the largest one, the price-leader, or the one with labour-relations leadership.

The wage-making process is internally concentrated on the key rates in the job clusters, these rates constituting the focal points for wage-setting forces among firms within the contour. In other words, the key wage rates in the job clusters are the channels of impact between the exterior developments (in the contour) and the interior rate structure of the firm.

In the short run, the anatomy of the wage structure of an internal market is determined by the level of technology, the administrative arrangements of firms, competitive patterns in product markets and the sources of labour supply. In the long run the crucial factors are the rate and pattern of industrialization, and changes in the supply of labour. One of the effects of industrial growth on large firms was the progressive elimination of personalized wages that existed in the early stages of industrial development, and the creation of explicit wage rates for defined occupations and jobs. The emergence of an explicit wage scale is linked to the structuring of the labour force by skill or performance, and it does not lend itself to a satisfying explanation of internal wage differentials by a simple demand and supply approach. The integrated production process creates a wide but variable area of joint demand for labour, while the better paying jobs appear at any given time to be rationed; therefore one job cannot be regarded as independent of related jobs, neither from the demand side nor from the supply one.

Finally, the process of economic development creates a hierarchy of wage rates on a national level exterior to an enterprise. This national wage structure may be influenced either by explicit rules or by the market forces. A greater

degree of interdependence in the internal wage structure of a firm is likely to emerge among firms in the same or closely allied product markets in a particular locality. This uniformity may spread to firms in more distant localities in the same product grouping or contour.

F. Wage Determination in Closed Internal Markets.

Do Neoclassical Theories Provide an Answer?

The assumptions implicitly made in the neoclassical wage theory are not valid in the wage determination of the internal labour market, the existence of fixed employment costs - regarding recruitment, screening and training - and the permanence of employment relations prevent the wage rate from being equal to the worker's marginal product. Doeringer and Piore¹⁵ argue that *"In sum, the forces which in neoclassical theory yield a determinate wage establish, in the internal market, only a series of constraints. The equality between the marginal product of labour and the wage of a job postulated by competitive economic theory is reduced to an equality between the discounted present value of expected cost and productivity streams calculated over the distribution of expected employment tenure for various groups within the enterprise."*

In short, dual theorists point out that the customary and formal wage administrative procedures, aiming to avoid bargaining and fixed labour costs inherent in internal markets, dictate certain solutions to the wage-setting problem; this solution, however, is achieved through a series of constraints, the most important ones being:

- a) The internal allocative administrative rules which define the shape of the internal mobility clusters.
- b) The rules related to efficiencies in the recruitment, screening and training.
- c) Custom.
- d) The relationship between earnings and social status.
- e) The individual utility functions.

Wachter¹⁶ argues that there is no infinite number of equilibrium internal wage structures but, contrary to the dual theory, management selects the wage structure that maximizes intrafirm efficiency.

According to Birnbaum¹⁷ earnings inequality may arise partly because differential experience in various jobs and job ladders offers individuals different earning opportunities for developing cognitive task skills, and partly because different job ladders change certain personality traits such as reliability, responsiveness to authority and discipline. In the internal markets, the earnings of those workers who have a strong preference for permanent employment relations and for jobs yielding a steady income are determined by their career origin and the promotional or on-the-job training opportunities attached to it, the much celebrated worker choice playing a marginal part in the long run.

Reder¹⁸ suggests that promotional ladders are institutional devices that serve as a mutual insurance scheme for both the firm and its employees. Employers seek to hedge hiring risks by promoting from within, while employees try to avoid risks related to their mathematical expectation of earnings during their lifetime. Furthermore, the author points out that the existence of Trade Unions and the external pressure of labour markets constrain the internal wage policies of the firms: we quote: *"In short, the very imperfect attempts at mutual insurance represented by a policy of promotion from within enormously complicate the relation between the value of the current marginal contribution of a productive agent and its reward, and alter the distribution of earnings in a variety of ways."*

The last shortcoming of neoclassical theories vis-à-vis wage determination in closed internal markets is their weakness to explain the fact that the large monopolistic firms, of which internal markets are mainly composed, pay higher wages to their employees than the ones dictated by the competitive market.*⁵ These firms being more vulnerable to the press seek to buy public approval and to stand well in public opinion. A large corporation, in addition, puts more emphasis on dependable and regimented workers, therefore it pays higher wages in order to avoid erratic performance on-the-job, excessive absenteeism, and to keep the "good will" of its employees. Finally, large corporations pay higher wages in order to dissuade workers from unionizing or, if strong unions do exist, to have friendly relations with them, thus maintaining their dependable work-force. Needless to say that large monopolistic firms have the potential to pass the higher wages on to the consumers.

Concluding, we note that the concept of the learning curve, the role of groups in the formation of individual preferences, custom and practise, Trade Unions, and the firm's internal and external status or relations are of critical importance to the distribution and growth of earnings in closed internal markets.

*⁵ Weiss¹⁹ found that concentrated industries pay high incomes for certain occupations, although the fact that these industries hire high quality labour may account for the evidence.

Dalton and Ford²⁰ recently found that even if we correct for the high quality of labour factor, there is still a positive relationship between concentration and high level of wages.

Masters²¹ presents empirical support to the hypothesis that plant size rather than concentration positively affects average wage rates.

Table

Employment Distribution by Type of Internal Labour
Market, 1965. (U.S.A)

TYPE OF LABOUR MARKET.	PERCENTAGE OF EMPLOYMENT		
<u>Structured</u>			81.4%
Enterprise type.		54.4	
Military services.	3.2		
Workers in public enterprises.	11.8		
Workers in institutions (includes hospitals, universities, museumns, etc.)	2.9		
Union workers in large enterprises.	11.8		
Non-union workers under industrial agreements.	1.3		
Workers outside the bargaining unit in large enterprises.	7.0		
Workers in large non-union enterprises.	7.0		
Workers in small enterprises.	27.0		
Craft type. (Workers in craft unions)		9.4	
<u>Unstructured.</u>			18.4
Proprietors and self-employed family workers.	12.0		
Farm laborers.	1.5		
Domestic Workers.	2.4		
Self-employed professionals in offices and laboratories.	1.6		
Workers performing odd jobs, service and repair work.	.9		
TOTAL			100.0

This table appears on page 42 of "Internal Labor Markets and Manpower Analysis" by Doeringer P. and Piore M. and it is derived from O.W.Phelps, "Introduction to Labor Economics, New York: McGraw-Hill Co., 1967.

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CHAPTER 4: "STOCHASTIC MODELS AND POINT PROCESSES"

We would like to remark that this chapter is not a contribution to the existing mathematical theory but it is intended to be a brief introduction to the theory of point processes and a guide to the non-mathematical reader.

A. Definition of a Stochastic Process.

Cox and Miller¹ define a stochastic process as a system which develops in time or space according to probabilistic laws, time being the most common parameter of the process. Massy et al² emphasize the ex ante probabilistic structure of the stochastic models, we quote, *"As we understand current terminology, a stochastic model is a model in which the probability components are built in at the outset rather than being added ex post facto to accommodate discrepancies between predicted and actual results. That is, the probabilistic components form an important part of the basic structure of the stochastic model."*

There have been a lot of criticisms related to the application of stochastic models into economics. The main criticism has been directed at the economic content of the models. Many economists^{*1} argue that the element of chance has become a substitute for economic theory, thus questioning or even rejecting the relevance of stochastic models as explanations of size distributions. Although the random elements are inherent in both the conditions and the assumptions under which the economic system has to function the stochastic models so far presented incorporate few of the familiar economic variables, and consequently these models set up to explain size distributions lack in economic content;

*1 See, for instance, Lydall³ and Sahota.⁴

we quote from Shorrocks⁵ "to some extent the problem is one of defining suitable stochastic analogues of deterministic theories, a problem which is aggravated when state variables, such as wealth classes, do not correspond to those used in deterministic models".

Let us try to clarify a basic misunderstanding that prevails among economists as far as the idea of randomness is concerned. It is a well-known fact in the literature that mathematical models are either deterministic or probabilistic (stochastic models falling under the latter category). In deterministic models we consider mathematical variables and the effect of any change in the system can be predicted with certainty. On the other hand, whenever the system is not fully specified or because of the unpredictable character of the human behaviour (and this is the case in economics) there is a certain uncertainty incorporated in any prediction or outcome, and so in this case we introduce random variables with probability distributions assigned to them. The introduction of random variables though does not necessarily mean that mere chance plays the important role in the system as many economists still believe. Mere chance or randomness, being the proper term for it, operates in a particular class of stochastic processes and it is directly related to the Markovian property and the lack of memory of the geometric and the negative exponential distributions, as we will explain later.

Bartholomew⁶ mentions a more general objection that it is often raised to the application of stochastic models in the social sciences. The argument that by allowing probability laws to govern human relations we deprive human beings of freedom of choice is not theoretically valid and, according to the author, it rests on a misunderstanding of the

nature of probability theory in model building; since human beings are free-decision makers, their behaviour is unpredictable and, therefore a probabilistic model is more appropriate to be applied, while a deterministic one would constrain human behaviour along a predetermined path. Bartholomew also makes a distinction between "explanatory" models and "black-box" ones. He defines the former as the ones that aim to explain the mechanism which governs the relationship between the input and the output variables of a certain system, while the latter merely state the relationship, a model of the regression type being a typical example.

B. The homogeneity Assumption and the Markovian Property.

Stochastic processes are basically divided according to two criteria, the assumption of homogeneity and the Markovian property, which have been the main obstacles to the introduction of economic theory into the stochastic models so far presented. Suppose we have a Markov process with discrete states in continuous time and let $X(t)$ denote the state occupied at time t . The process is in principle defined when we have a set of functions (transition probability functions) $p_{ij}(t)$ with the interpretation: $p_{ij}(t) = \text{probab } \{X(t+u) = j | X(u) = i\}$ and the process is time-homogeneous if the probability functions are independent of u . The homogeneity assumption has enabled economists to make extensive use of the properties of ergodic processes and equilibrium aspects of the systems; but if the parameters of the system are functions of economic variables, generally time-dependent, the time-homogeneous models cannot by nature explain the size distributions of income or wealth.

Shorrocks⁷ criticized the time-homogeneous models analytically and he introduced a non-homogeneous birth and death process with "immigration" in order to explain the size distribution of wealth, the transition intensities of the process being dependent upon the individual's age t .

Although representing the process parameters as functions of time-dependent economic variables seems to be a formidable task, it does not give a satisfactory answer to the objections raised so far; as Quandt⁸ in his model of the size distribution of firms points out, we quote, "*what guarantee or what reason for belief does one have that the transition matrix in one industry will be the*

same, or nearly the same as in another? In general one cannot assume the inter-industry stability of transition matrices. If one is willing to hypothesize, contrary to what some have held, that cost and demand functions have something to do with the manner in which industries develop, one may arrive at a model of industry development in which transition matrices depend on the following factors:

- 1) The nature of the short-run cost function.....
- 2) The nature of the long-run cost function.....
- 3) The nature of oligopolistic arrangements - or the absence thereof - in a given industry
- 4) The general configuration of competing products, changes in relative technology, and changes in relative demands.

But it is not even plausible to suggest that only the values of certain parameters will be different; in all likelihood the nature of the stochastic process itself will differ from case to case. Accordingly it would be surprising if the same distribution (with different estimated values for the parameters) were to fit all cases".

Ruggles⁹ emphasizes the fact that a wide range of institutional, historical and sociological factors are directly related to the overall distribution of incomes, while Shorrocks¹⁰ says the following, "The Markov property implies that individuals with the same income will have identical predictions for the future, regardless of whether in the recent past their income has been increasing or falling, and it denies the possibility of serial correlation in the growth rates for different periods. This is incompatible with Friedman's distinction between permanent

and transitory income which predicts a negative correlation between growth rates of income".

Consequently, so far as the application of stochastic processes into economics is concerned it seems that the Markovian property plays an important role.

The Markovian property can be described as follows: suppose we have a discrete time stochastic process $\{X(t), t=0,1,2,\dots\}$ or a continuous parameter one $\{X(t), t \geq 0\}$, we call the process Markovian if, for any set of n time points $t_1 < t_2 < \dots < t_n$, the conditional distribution of $X(t_n)$, for given values of $X(t_1), X(t_2), \dots, X(t_{n-1})$, depends on $X(t_{n-1})$, the most recent known value. In other words $\text{prob.}\{X(t_n) \leq x_n | X(t_1) = x_1, \dots, X(t_{n-1}) = x_{n-1}\} = \text{prob.}\{X(t_n) \leq x_n | X(t_{n-1}) = x_{n-1}\}$ and so the present of the process determines its future, the past being of no importance. However, we can examine the property of the same process not by studying the random variable $X(t)$ but by considering the points of time where transitions occur; in order to examine the latter approach we have to introduce a particular class of stochastic processes, the point processes.

C. Point Processes and Their Applications.

Point or counting^{*2} processes are a general type of non-Markovian process. In a point process there are certain events which possess the regenerative quality, that is once a regenerative point has been reached, the whole development preceding and leading up to this point (or event) is of no further consequence for the following process. The regenerative event embodies the whole past, that is any information relevant for the further development of the process. More precisely, suppose that a process $\{X(t), t \geq 0\}$ is such that for some particular time T and for all $t > T$ the conditional probability distribution of $X(t)$, given $X(T)$, is equal to the conditional probability distribution of $X(t)$, given $X(\tau)$, for all $\tau \leq T$, the point T or the event by which it can be identified is called a regeneration point for the process and any process that has such points is called a counting or point process. Markov processes are those in which every point along the time-axis has the regenerative quality, thus a point process being a generalization of the Markov one.

A very important category of point processes is the well-known renewal process; an integer-valued process $\{n(t), t \geq 0\}$ corresponding to a series of points or events distributed on the interval $(0, t)$ is called a renewal process if the inter-arrival times τ_1, τ_2, \dots between successive events are independent identically distributed positive random variables. The proper random variable $n(t)$ is the number of renewals or events during the interval $(0, t)$, and it has finite moments of all orders.

Let S_n be the waiting time to the n th event, that is the time it takes for n events to occur. The successive

^{*2} As Parzen¹¹ refers to them.

inter-arrival times are defined as follows:

τ_1 is the time from zero to the first event and for $i > 1$,

τ_i is the time from the $(i-1)$ st to the i th event.

Therefore $S_n = \tau_1 + \tau_2 + \dots + \tau_n$ for $n \geq 1$. There is a basic relation between a point process $\{n(t), t \geq 0\}$ and the corresponding sequence of waiting times $\{S_n\}$. For $t > 0$ $n(t) = \max \{n | S_n \leq t\}$, or $n(t) \leq n$ if and only if $S_{n+1} > t$ $n = 1, 2, \dots$. It follows that $n(t) = n$ if and only if $S_n \leq t$ and $S_{n+1} > t$.

The Markovian property is uniquely related to a particular renewal process, the Poisson process. Let $n(t, t + \Delta t)$ denote the number of events in the interval $(t, t + \Delta t)$. Suppose that, for some positive constant ρ , as $\Delta t \rightarrow 0$, we have:

$$\text{prob.}\{n(t, t + \Delta t) = 0\} = 1 - \rho(\Delta t) + o(\Delta t), \quad (1)$$

the last term implying that the probability of more than one events occurring simultaneously is zero in the interval Δt .

$$\text{Also prob.}\{n(t, t + \Delta t) = 1\} = \rho(\Delta t) + o(\Delta t), \quad (2)$$

therefore $n(t, t + \Delta t)$ is completely independent of occurrences in the interval $(0, t]$. We call this stochastic series of events a Poisson process of rate ρ . Let us now consider a new time origin at t_0 , which may be a point at which an event has just occurred, or any fixed point. Let $t_0 + z$ be the time of the first event after t_0 , and let us calculate the probability distribution of the random variable z . If $P(x) = \text{prob.}(z > x)$ it follows that

$$P(x + \Delta x) = \text{prob.}(z > x) \text{ times the prob.}\{\text{no event occurring in } (t_0 + x, t_0 + x + \Delta x) | z > x\} \quad (3)$$

Equation (3) holds for any renewal process but a special feature of the Poisson process that describes the Markovian

property is that the conditional probability in (3) is not affected by the condition $z > x$, which refers to what happens at or before $t_0 + x$. Therefore all the properties of the Poisson process referring to its future behaviour, after t_0 , are independent of what happens at or before t_0 . In this case we can replace the conditional probability in (3) with the unconditional one in (1) so that (3) becomes

$$P(x + \Delta x) = P(x)\{1 - \rho(\Delta x) + o(\Delta x)\}, \text{ or}$$

$$P'(x) = -\rho P(x). \quad (4)$$

The solution of equation (4) is $P(x) = P(0)e^{-\rho x}$, with the initial condition $P(0) = \text{prob.}(z > 0) = 1$. Finally we obtain $P(x) = e^{-\rho x}$, and the probability density function of z is $\rho e^{-x\rho}$, $x > 0$, that is the negative exponential density with parameter ρ . Since \underline{x} is a random variable corresponding to the time between one event and the next one, the probability that the waiting time until the next event is longer than \underline{x} is $e^{-\rho x}$, the exponential law for the waiting time being the only continuous distribution said to be endowed with complete "lack of memory"¹²; in other words, the probability that we have to wait \underline{x} time units for the next event is independent of the time we have already waited since the last event occurred. In the discrete case the role played by the exponential distribution is assumed by the geometric one.

Summing up, the Poisson process is fully determined by the negative exponential law and events occurring in the process are referred to as events occurring at random. While in statistics the word "random" indicates independence of observations, the term here denotes that the Poisson process assumes independence of occurrence of events so

that an infinite number of points (events) is randomly distributed over the interval $[0, \infty)$. Any Markov process, as we noted earlier on, is characterized by the exponential (or the geometric) law in the sense that the underlying point process, the process counting the number of transitions, is of the Poisson type.

Point, or more specifically, renewal processes have been increasingly applied to the various branches of the social sciences. Thus, Coleman¹³ emphasized the importance of the Poisson process in describing various problems in sociology, and Bartholomew¹⁴ thoroughly examined the application of renewal processes to the turnover of people in an organization; finally, Massy et al¹⁵ introduced a non-homogeneous Poisson model where purchasing decisions made by individual families are represented as discrete events occurring across the time axis. Nevertheless, the role of point processes in economics has never been examined before. Only Steindl¹⁶ tried to draw attention to the role of these processes, we quote, *"We have obvious regeneration points in the pay-days when wages and salaries are received, or in the accounting periods of firms. Moreover, all consumption is a renewal process, as also is production"*.

Although one can hardly believe that economic phenomena can be necessarily described by a Markov process, the known models representing stochastic processes that generate distributions of economic variables have the Markov property. This may be due to the existing elegant mathematical theorems of the Markov processes, while point processes are difficult to handle. The relevance of the negative exponential law in the underlying point processes has never been questioned. It may be true that this assumption reasonably describes

actual phenomena such as the length of the telephone conversations within a city, or the duration of machine repairs. But when it comes to size distribution of economic variables, such as income or earnings, it is not at all certain whether the "lack of memory" assumption is empirically supported or not, this particular area has been completely unexplored.

As we will discuss in detail in the following chapter, the distribution of earnings of an individual in a large bureaucratic corporation cannot be merely represented or generated by the Poisson process. If we only apply this process then the employee's earnings are likely to change at every point of his length-of-service; the probability of the time elapsing until the next change of his earnings is independent of the time already elapsed since the last change occurred. Obviously, only at certain points of the individual's length-of-service a change of his earnings takes place and this is related to the wage structure which the individual is subject to.

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We will examine the course of evolution of an employee's earnings (in money terms) within the framework of point (stochastic) processes. According to the theory developed for those processes, we have to consider events occurring at random points of the time axis. Let us call each positive change in the individual's earnings an event that occurs across his length-of-service.

As Doeringer and Piore¹ have pointed out, an income increase may be related to a promotion to the next higher job grade in the job structure (vertical differentiation of the wage structure), or it may be identified with an increase due to meritorious individual performance in the same job level (horizontal differentiation of the wage structure). Demotions, or negative income changes, are excluded.*¹

Each one of the income increases may correspond to a particular amount of experience (general or specific) which may be associated with the skills and responsibility required for a job.*² Each event increases the employee's earnings, according to a functional equation characterizing each firm (or industry). The income increment could be a random

*¹

The assumption not perhaps an important one, since - generally speaking - if an employee was faced with a negative income increase he would prefer to leave the firm. According to Reder², employers try to avoid cutting wages due to the employee's emotional resistance, unions playing a vital role in this respect. Firms pursue other means of reducing labour cost per unit of output in a depressed labour market, such as imposing more selective methods in their hiring requirements or increasing the standards of the various job assignments.

*²

This is reminiscent of Lydall's R factor³ (responsibility, or hierarchy effect).

variable independent of the point process as, perhaps, with the self-employed person. We make here, however, the assumption that the increment is constant on a logarithmic scale and the following equation exists:

$$y = y_0 e^{an} \quad (1)$$

where y is the employee's earnings, $n = 0, 1, 2, \dots$, are the point events (or income changes), and a being a parameter, is the increment. When $n = 0$, we have $y = y_0$, which is the employee's initial income value as soon as he enters the firm, with his length-of-service being equal to zero. Therefore, the firm considered is characterized by an exponential growth of individual earnings. This does not necessarily apply to any other firm, where we might have a different functional relationship (e.g. a linear growth of earnings). Instead of considering one firm (or a bureaucratic corporation in the internal labour market), we could of course refer to any wage structure across firms, characterized by the same promotion policy and pattern of mobility.

Let us consider two general cases (A and B) concerning the transition (or promotion) rates from one income level to the next; in case A, we will make the assumption that the lengths of stay in each income level are independently and identically distributed positive random variables, following an arbitrary probability density function. In case B, we will drop this assumption and we will assume some form of interdependence between the lengths of stay.

As far as case A is concerned, we will be examining two alternative possibilities:

- I. The first possibility is when transitions depend upon seniority within each income level and the transition rate (or the hazard function)⁴ is an increasing or

decreasing function of the length of stay at the income level;

- II. The second possibility is the Markovian model when the transition rate is constant or rather, independent of seniority within each income level.
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A. THE INDEPENDENCE ASSUMPTION OF THE LENGTH-OF-STAY

According to the theory of point processes, any arbitrary promotion rate is uniquely determined by the probability density function of the continuous and positive random variables τ_i corresponding to the length-of-stay (or seniority) in the income levels $i = 1, 2, 3, \dots, n$. Assuming that the τ_i random variables are independently and identically distributed with a p.d.f. $f(\tau)$, the time it took an employee to reach the n^{th} income level is the random variable S_n . Clearly, $S_n = \tau_1 + \tau_2 + \dots + \tau_n$ and its probability density function is given by $p(S_n) = f^{*n}(\tau)$, which is the n^{th} convolution of $f(\tau)$ with itself.

The following are well-known results in the theory of point processes:

Let $P_n(s) = \text{Prob.}\{S_n \leq s\}$, and $\pi_n(s) = \text{Prob.}\{n(s) = n\}$, that is, the probability of attaining the n^{th} income level within \underline{s} years of service. Then, $n(s) \geq n$ if and only if $S_{n+1} \leq s$. Therefore

$$\pi_n(s) = \text{Prob.}\{n(s) \geq n\} - \text{Prob.}\{n(s) \geq n+1\} = P_n(s) - P_{n+1}(s) \quad (2)$$

The above probabilities correspond to the distribution of the number of income changes, or promotions, in a random sample of employees of equal length-of-service.

We have now to consider the probability mass function of the number of promotions in a mixed sample of employees heterogeneous in the length-of-service. In order to do that, we will treat the length-of-service as a continuous random variable and derive its probability density function according to certain assumptions regarding the rate of entry and exit in the firm. Then we will integrate over all possible cohorts

of employees with different lengths-of-service, thus obtaining the stable probability mass function of the number of promotions in a mixed sample of employees.

It would be likely that the rate of exit depends upon two factors: first, it is an increasing function of the individual's age, therefore the probability of retirement or death increases with age. The second factor is related to the employee's discontent - if he thinks he is not satisfactorily remunerated for his services. So the rate of exit may also be a decreasing function of the number of income changes or promotions and, consequently, a decreasing function of the employee's length-of-service.^{*3} It may be that the length-of-service is the only factor critically important to the rate of exit, but it is not easy to obtain an explicit result with any arbitrary function of the length-of-service due to the integration problems that appear when we consider a mixed sample of employees. Since one of the main functions of the internal labour market is to reduce labour turn-over, and since there is a strong preference for permanent employment connections by both employers and employees, we will assume that the rate of exit is constant. The constancy of the exit rate assumption implies that the stochastic process from which the probability density function of the length-of-service is generated, is of the Markovian character.

Taking into account that the rate of exit is a constant, $h(s) = m$, the survival probability, that is the probability

^{*3} Age may be a function of the length-of-service: A.Alexander⁵ has argued that firm experience measured in years of length-of-service, when entered explicitly, reduces sharply the impact of age upon mobility.

of an employee staying in the firm at least \underline{s} years

will be:

$$\gamma(s) = e^{-\int_0^s h(x)dx} = e^{-ms} \text{ for } 0 < s < \infty \quad (3)$$

As the length of service \underline{s} tends to zero the survival probability tends to one, so the boundary condition will be $\gamma(0) = 1$.

In order to simplify further the mathematical computation, we assume that there is a constant rate of entry per employee already in the firm, so that the number of individuals who enter at time T is proportional to the existing number of employees at time T . The number of employees with length-of-service \underline{s} years is determined by the number of employees who entered the firm \underline{s} years ago times the probability of staying in the firm at least \underline{s} years, that is

$$v(s, T) = \omega(T-s) \gamma(s), \text{ if } s < T.$$

Consequently, the existing number of employees at time $T, K(T)$, is given by the equation

$$K(T) = N_0 \gamma(T) + \int_0^T \omega(T-\tau) d\gamma(\tau),$$

where N_0 is the initial population at time $T=0$. If the rate of entry per employee is \underline{b} , we have $\omega(T) = bK(T)$, $\omega(T)\delta T$ being the expected (mean) number of individuals who enter the firm in a small interval near T . Making the substitution in the above expression we obtain

$$\omega(T) = N_0 b \gamma(T) + b \int_0^T \omega(T-\tau) d\gamma(\tau) \quad \dots (4)$$

In essence, $\omega(T)$ corresponds to the renewal density and equation (4) is the integral equation of renewal theory with $b\gamma(T)$ being a defective distribution. In the context of the renewal theory, the lengths-of-service \underline{s} of all employees belonging to a particular firm are independently distributed random variables, with identical frequency distributions $\gamma(s)$.

If we write the Laplace transform of $\phi(u) = b \int_0^{\infty} e^{-us} d\gamma(s)$, the Malthusian parameter for $b\gamma'(s)$ is the root, provided it exists, of the equation:

$$\phi(\varepsilon) = b \int_0^{\infty} e^{-\varepsilon s} d\gamma(s) = 1, \text{ or } \phi(\varepsilon) = bm \int_0^{\infty} e^{-\varepsilon s} \cdot e^{-ms} ds$$

for which we obtain $\varepsilon = m(b - 1)$... (5)

If the rate of entry per individual, \underline{b} , is greater than one, then the net rate of increase of the population of employees, the Malthusian parameter $\underline{\varepsilon}$, is positive and the population increases. If the entry rate equals one, then the Malthusian parameter is zero and the population remains the same. When these cases occur, the Malthusian parameter always exists.

Equation (4) can be solved by means of the Laplace transforms, but before we proceed with the solution we will derive the asymptotic estimate for $\omega(T)$. Provided that the population of employees in the firm is non-decreasing ($\varepsilon \geq 0$), $\gamma(s)$ is not a Lattice^{*4} (arithmetic) distribution and for T sufficiently large, the solution of the renewal equation (4) approaches a limit (renewal density theorem).

Therefore $\omega(t) \rightarrow \frac{C e^{\varepsilon T^{*5}}}{\phi'(\varepsilon)}$, where

$$\phi'(\varepsilon) = b \int_0^{\infty} s e^{-\varepsilon s} d\gamma(s) = \frac{1}{bm}, \text{ and } C = \int_0^{\infty} \text{Nob} \gamma(T) e^{-\varepsilon T} dT = \frac{\text{No.}}{m}.$$

Finally, $\omega(T)$ tends to $\text{Nob} e^{m(b-1)T}$.

*4

A discrete distribution of a random variable X is called a lattice distribution if there exist numbers \underline{a} and \underline{b} (both positive) such that every possible value of X can be written in the form $a + kb$, where k assumes integer values.

*5

See W. Feller⁶ and Th. E. Harris.⁷

Since $\gamma(s) = e^{-ms}$, equation (4) can be written as follows:

$\omega(T) = \text{Nob } e^{-mT} + bm \int_0^T \omega(T - \tau) e^{-m\tau} d\tau$, by taking the Laplace transforms of both sides we obtain

$\omega^*(u) = \frac{\text{Nob}}{m+u} + bm \frac{\omega^*(u)}{m+u}$, where $\omega^*(u)$ is the Laplace transform of $\omega(T)$, $(m+u)^{-1}$ is the Laplace transform of $e^{-m\tau}$ and $\frac{\omega^*(u)}{m+u}$ represents the convolution of $\omega(T - \tau)$ and $e^{-m\tau}$. By solving for $\omega^*(u)$ we obtain $\omega^*(u) = \frac{\text{Nob}}{(m - bm) + u}$ and by inverting, we derive the solution, $\omega(T) = \text{Nob } e^{m(b-1)T}$, which coincides with the asymptotic result.

If a steady-state equilibrium of the length-of-service distribution has been established, the number of employees with length-of-service \underline{s} years at time T approaches (for $T - s$ sufficiently large in the case of the asymptotic estimate):

$$v(s, T) \rightarrow \text{Nob } e^{m(b-1)(T-s)} e^{-ms} = \text{Nob } e^{m(b-1)T} e^{-mbs}.$$

In order to find the density function of the length-of-service distribution, we have to divide the number of employees with length-of-service \underline{s} years, $v(s, T)$, by the number of all employees existing in the firm at time T . Therefore we have

$$\frac{v(s, T)}{\int_0^\infty v(s, T) ds} = \frac{\text{Nob } e^{m(b-1)T} e^{-mbs}}{\int_0^\infty \text{Nob } e^{m(b-1)T} e^{-mbs} ds} = bm e^{-bms} \quad (6)$$

By combining (2) and (6) and integrating over all possible values of the length-of-service \underline{s} , we derive the probability mass function of the number of promotions (or income changes) in a mixed sample of employees, that is:

$$\pi_n = \int_0^\infty \pi_n(s) bm e^{-bms} ds, \quad \text{or}$$

$$\pi_n = bm \left\{ \int_0^\infty P_n(s) e^{-bms} ds - \int_0^\infty P_{n+1}(s) e^{-bms} ds \right\} \quad (7)$$

The above integrals are, in essence, the Laplace transforms of the distributions $P_n(s)$ and $P_{n+1}(s)$ for bm . These Laplace transforms can be evaluated in terms of the probability density function of seniority in each income level as follows:

$$\int_0^{\infty} P_n(s) e^{-bms} ds = \frac{\Lambda(bm)^n}{bm}, \text{ where } \Lambda(bm) \text{ is the Laplace transform of the density function } p(S_n). \text{ But } p(S_n) = f^{*n}(\tau), \text{ and since } f^{*n}(\tau) \text{ represents a convolution we have } \Lambda(bm) = \sigma^n(bm), \text{ that is the Laplace transform of } f(\tau) \text{ raised to the } n^{\text{th}} \text{ power (provided it exists). Finally, we obtain } \pi_n = \sigma^n(bm) - \sigma^{n+1}(bm). \quad (8)$$

and by substituting (1) into (8) we derive the probability of an employee of any length-of-service having earnings, y , which is

$$\pi_y = \sigma^{\frac{1}{a}} \ln\left(\frac{y}{y_0}\right) (bm) - \sigma^{\frac{1}{a}} \ln\left(\frac{y}{y_0}\right) + 1 (bm). \quad (9)$$

I An application where the promotion rate is an increasing (or decreasing) function of seniority within each income level. (The non-Markovian case).

Suppose that the τ_i random variables follow the gamma density function with parameters θ and λ , that is:

$f(\tau) = \frac{\lambda^\theta}{(\theta-1)!} \tau^{\theta-1} e^{-\lambda\tau}$ for $\tau > 0$ and $\theta, \lambda > 0$. In this case, the promotion rate is an increasing function of seniority within the income level if $\theta > 1$, and it is a decreasing one if $0 < \theta < 1$.⁹ The Laplace transform of $f(\tau)$ for bm is $(\frac{\lambda}{\lambda+bm})^\theta$ and substituting into (9), we obtain the probability mass function of earnings in a mixed sample of employees:

$$\begin{aligned} \pi_y &= \left(\frac{\lambda}{\lambda+bm}\right)^{\frac{\theta}{a} \ln(\frac{y}{y_0})} - \left(\frac{\lambda}{\lambda+bm}\right)^{\frac{\theta}{a} \ln(\frac{y}{y_0}) + \theta} \\ &= \left\{1 - \left(\frac{\lambda}{\lambda+bm}\right)^\theta\right\} \left(\frac{\lambda}{\lambda+bm}\right)^{\frac{\theta}{a} \ln(\frac{y}{y_0})} \end{aligned} \quad (10)$$

We can modify the above expression by letting

$\ln(1 + \frac{bm}{\lambda}) = c$, so that $1 + \frac{bm}{\lambda} = e^c$, and $\frac{\lambda}{\lambda+bm} = e^{-c}$. Then (10) becomes

$$\pi_y = (1 - e^{-c\theta}) \left(\frac{y_0}{y}\right)^{\frac{c\theta}{a}}, \quad (11)$$

which is the discrete form of the Pareto density. The Pareto coefficient $\frac{c\theta}{a}$ depends upon the rate of entry and exit in the firm, on the constant income increment a (on a logarithmic scale), and on the parameters θ and λ of the gamma density.

II An application with a constant promotion rate.

(The Markovian case)

When the promotion rate is independent of seniority within each income level, the random variables τ_i follow the negative exponential density with expectation $\frac{1}{\lambda}$ that is $f(\tau) = \lambda e^{-\lambda\tau}$. The Laplace transform of $f(\tau)$ for bm is $\frac{\lambda}{\lambda + bm}$ and following exactly the same procedure, we derive again the Pareto probability mass function, the final expression of the probability mass function of earnings being:

$$\pi_y = (1 - e^{-c}) \left(\frac{y_0}{y} \right)^{\frac{c}{a}} \quad (12)$$

N.B.: We could assume that the promotion rate of the promotion process λ is a function of either the rate of entry or exit without this affecting the form of the final result apart from the Pareto coefficient, which has to be adjusted accordingly.

B THE INTERDEPENDENCE OF THE LENGTH-OF-STAY.

Suppose now that the times between transitions, that is, the length-of-stay in each income level $\tau_1, \tau_2, \dots, \tau_n$ are not independent random variables. We can distinguish and examine both a probabilistic and a deterministic interdependence. Let us assume that each transition time depends upon the previous one according to a simple (linear) functional relation. Let τ_1 follow an arbitrary distribution and let $\tau_i = d\tau_{i-1}$, where $d > 0$ is a constant. Then $\tau_2 = d\tau_1$

$$\begin{aligned} \tau_3 &= d\tau_2 = d^2\tau_1 \\ \tau_n &= d\tau_{n-1} = d^{n-1}\tau_1 \end{aligned}$$

The sum $S_n = \tau_1 + \tau_2 + \dots + \tau_n$, the time it takes an employee to reach the n^{th} income level, is equivalent to

$S_n = \tau_1 + d\tau_1 + d^2\tau_1 + \dots + d^{n-1}\tau_1 = \tau_1(1 + d + d^2 + \dots + d^{n-1})$. But $(1 + d + d^2 + \dots + d^{n-1})$ is the sum of a geometric series and it is equal to $\frac{1 - d^n}{1 - d}$.

Therefore $S_n = \frac{(1 - d^n)\tau_1}{1 - d}$ and the problem is reduced to a simple transformation. If the continuous random variable τ_1 follows the probability density function $f(\tau_1)$ then

$$p(S_n) = \frac{(1 - d)}{(1 - d^n)} f\left\{\frac{(1 - d)S_n}{1 - d^n}\right\}. \quad (13)$$

As in the independence assumption case, the probability to attain the n^{th} income level within \underline{s} years of service will be: $\pi_n(s) = \text{prob. } \{n(s) \geq n\} - \text{prob. } \{n(s) \geq n + 1\} = P_n(s) - P_{n+1}(s)$, where $P_n(s) = \text{prob. } \{S_n \leq s\}$. Finally, in a mixed sample of employees, the probability mass function of the number of income changes will be

$$\pi_n = \left(\frac{1 - d}{1 - d^n}\right) \sigma_1(bm) - \left(\frac{1 - d}{1 - d^{n+1}}\right) \sigma_2(bm), \quad (14)$$

where $\sigma_1(bm)$ and $\sigma_2(bm)$ are the Laplace transforms of

$f\{\frac{(1-d)s}{1-d^n}\}$ and $f\{\frac{(1-d)s}{1-d^{n+1}}\}$ respectively for bm. By

substituting $\frac{1}{a} \ln (\frac{y}{y_0})$ for \underline{n} we obtain the probability mass function of earnings \underline{y} in a mixed sample of employees.

APPLICATION

Let us assume for instance, that the probability of an employee attaining the first income level is independent of seniority, that is the transition time τ_1 follows the negative exponential density with expectation λ^{-1} .

Then (13) becomes $p(S_n) = \frac{(1-d)\lambda}{1-d^n} e^{-\frac{(1-d)}{1-d^n} s}$ and the probability mass function of the number of promotions in a mixed sample of employees is:

$$\pi_n = \lambda(1-d) \{ \lambda(1-d) + bm(1-d^n) \}^{-1} - \lambda(1-d) \{ \lambda(1-d) + bm(1-d^{n+1}) \}^{-1}$$

$$\text{and, therefore, } \pi_y = \lambda(1-d) \{ \lambda(1-d) + bm(1-d^{\frac{1}{a}} \ln(\frac{y}{y_0})) \}^{-1} - \\ - \lambda(1-d) \{ \lambda(1-d) + bm(1-d^{\frac{1}{a}} \ln(\frac{y}{y_0}) + 1) \}^{-1}. \quad (15)^{*6}$$

The second case arises if we assume a serial probabilistic dependence between the τ_i 's. For instance, the stochastic equivalent of the first (deterministic) case is a first-order autoregressive process in which the transition times obey the law of proportionate effect. Let $\ln \tau_i = \ln \tau_{i-1} + \ln d_i$, where the set $\{\ln d_i\}$ is independent of the set $\{\ln \tau_i\}$. Assuming that the $\{\ln d_i\}$ are independently and identically distributed, following the normal distribution, then for a large number of promotions, the τ_i 's tend to be log normally distributed. Since the Laplace transform of the log normal distribution cannot be easily found, this case does not lead to an explicit result.

Instead, let us assume that $\tau_i = \tau_{i-1} + d$, where the d s are positive and continuous random variables identically and independently distributed with a probability density function $\xi(d)$ for $0 < d < \infty$. Then $\tau_n = \tau_1 + (n-1)d$, and the sum

$$S_n = \tau_1 + \tau_2 + \dots + \tau_n = n\tau_1 + \frac{n(n-1)d}{2}. \quad \text{Suppose now that } \tau_1 \text{ is a random variable, independent of } d, \text{ following an arbitrary}$$

*6

See Appendix at end of Chapter.

density function $f(\tau_1)$. If we put $n\tau_1 = T_1$ or $\tau_1 = \frac{T_1}{n}$, the probability density function of T_1 will be $\frac{1}{n} f(\frac{T_1}{n})$. Also by putting $\frac{n(n-1)d}{2} = D$, we obtain the p.d.f. of D which is

$\frac{2}{n(n-1)} \xi\{\frac{2}{n(n-1)}D\}$. The probability density function of S_n is the convolution of the two density functions and passing into the Laplace transforms for bm we derive the Laplace transform of $p(S_n)$, that is

$\Lambda(bm) = \frac{2}{n^2(n-1)} \sigma_{T_1}(bm, n) \sigma_D(bm, n)$, where $\sigma_{T_1}(bm, n)$ and $\sigma_D(bm, n)$ are the Laplace transforms of $f(\frac{T_1}{n})$ and $\xi\{\frac{2}{n(n-1)}D\}$ respectively.

Finally, the probability mass function of the number of promotions in a mixed sample of employees will be:

$$\begin{aligned} \pi_n &= \frac{2}{n^2(n-1)} \sigma_{T_1}(bm, n) \sigma_D(bm, n) - \\ &\quad - \frac{2}{n(n+1)^2} \sigma_{T_1}(bm, n+1) \sigma_D(bm, n+1) \end{aligned} \quad (16)$$

and by substituting $\frac{1}{a} \ln(\frac{y}{y_0})$ for \underline{n} , we derive the probability mass function of earnings.

C. THE INCOME DISTRIBUTION IN THE INDUSTRY.

The income distribution in an industry will be the weighted sum of income distributions of individual firms, assuming a unique wage structure for each firm. Alternatively, if we have different wage structures across firms, the aggregate income distribution will be the weighted sum of the income distributions derived from each existing wage structure. The weights attributed are related to the number of individuals employed in the firm, expressed as a proportion of all individuals employed in the industry.

According to the length-of-service process we have presented the number of workers employed in the firm j at time T is

$$\int_0^{\infty} v(s, T) ds = \int_0^{\infty} No_j b_j e^{m_j(b_j - 1)(T - s)} e^{-m_j s} ds = \frac{No_j}{m_j} e^{m_j(b_j - 1)T},$$

where No_j is the initial employee population of the firm j at time $T = 0$ and m_j , b_j are the rate of exit and the rate of entry per employee respectively. Therefore, the weight attributed to firm j is

$$p_j = \frac{(m_j)^{-1} No_j e^{m_j(b_j - 1)T}}{\sum_{j=1} (m_j)^{-1} No_j e^{m_j(b_j - 1)T}}$$

If all firms in the industry experience the same entry and exit rates, then the weights are independent of both the time T and the entry and exit rates, that is $p_j = \frac{No_j}{\sum No_j}$.

Finally, the aggregate income distribution in the industry will be:

$$\Pi_Y(T) = \sum_{j=1} p_j \pi_{y_j} \quad (17)$$

D. CONCLUSION.

The present chapter was concerned with the stochastic approach towards the problems of income differentials of employees in bureaucratic organizations which characterize internal labour markets. The most striking features of these markets are that employees are organized in various hierarchic grades, or job levels; that the probability of moving from one grade, with its defined salary, to the next, depends in part on the length of time that the employee has been in that particular grade.

Basically, we presented a non-Markovian model which dealt with the distribution and growth of earnings of employees within a firm. At a later state, we considered the aggregation problem, that is, the income distribution phenomenon across firms, or wage structures, remarking that an industry may consist of firms with different wage structures.

While presenting certain applications of the general model, our aim has not been to derive a specific income distribution - different assumptions lead to different results - but to derive conclusions concerning the inequality of earnings within a wage structure. For example, we have shown that under the hypothesis of equidistant income increments on a logarithmic scale, the assumptions we made regarding the promotion rate of employees in our model can generate the Pareto's law with the Pareto coefficient controlling all the known measures of inequality. Since the above coefficient depends upon the parameters θ and λ of the gamma function, the rate of entry and exit and the income increment, the following basic conclusions can be drawn from the model. First, by increasing either the rate of entry or the rate of exit, we

reduce the inequality of earnings. Second, by increasing the income increment, we increase the existing inequality. Thirdly, we have less inequality of earnings when the promotion rate is an increasing function of seniority within the income level ($\theta > 1$) and more inequality when the promotion rate is a decreasing function of seniority ($0 < \theta < 1$), the Markovian case ($\theta = 1$) being the middle point.

So far as the industry level is concerned the inequality of earnings depends upon the existing inequality in each individual firm or wage structure and upon the size of each firm, the size being represented by the number of employees working for the firm; thus, the larger the firms with a low degree of inequality the smaller the value of the aggregate measure of the inequality of earnings.

APPENDIX TO CHAPTER FIVE.

The probability mass function of equation (15) is one of the three highly skewed distributions derived in the thesis, apart from the Pareto one. An explicit solution for the statistical moments of the functions is extremely difficult to be obtained, instead we have estimated the mean and the second and third moments about the mean from calculated values based on a particular set of parameter values.

For the three probability mass functions we have obtained two kinds of comparative results.

First, with respect to each individual function we have estimated the effect of a change in the value of each parameter on the relative skewness by holding that of the others constant. We assume that the relative skewness

$$Sk_r = \frac{\pi_3}{\frac{\pi_2^{\frac{3}{2}}}{2}},$$

where π_2 and π_3 are the variance and the absolute skewness respectively (second and third moment), is some measure of inequality in the distribution of labour incomes.

Secondly, we have provided some basis for a cross distribution comparison of the relative skewness, given that the means of the three distributions are roughly equal. The equality of the means is derived by allocating the same value to all common parameters and a specific set of values to the parameters particular to each distribution.

So far as the latter case is concerned, the numerical values and a graphic representation of each distribution are being provided for annual incomes ranging from 3,000 to 15,600 pounds. The width of each income class is assumed to be 300 pounds.

By letting $d = e^c$ for $d \neq 1$, equation (15) can be written in the following form:

$$\pi_y = \lambda(1 - e^c) \left[\frac{1}{\lambda(1 - e^c) + bm \left[1 - \left(\frac{y}{y_0} \right)^{\frac{c}{a}} \right]} - \frac{1}{\lambda(1 - e^c) + bm \left[1 - \left(\frac{y}{y_0} \right)^{\frac{c}{a}} \right]} \right] .$$

Computer calculations have shown that by increasing either the rate of exit or entry or the income increment, the relative skewness of the above p.m.f. increases. On the other hand, relative skewness decreases as either the initial income or the parameter, λ , of the probability distribution τ_1 or the parameter, c , and consequently the constant promotion time increment d , increase.

Table.

When $b = 1$ (constant population of employees),
rate of exit $m = 0.2$, initial income $y_0 = 3,000$, $c = 1.4$,
 $\lambda = 0.5$ and the income increment $a = 0.1$ the following values
have been obtained:-

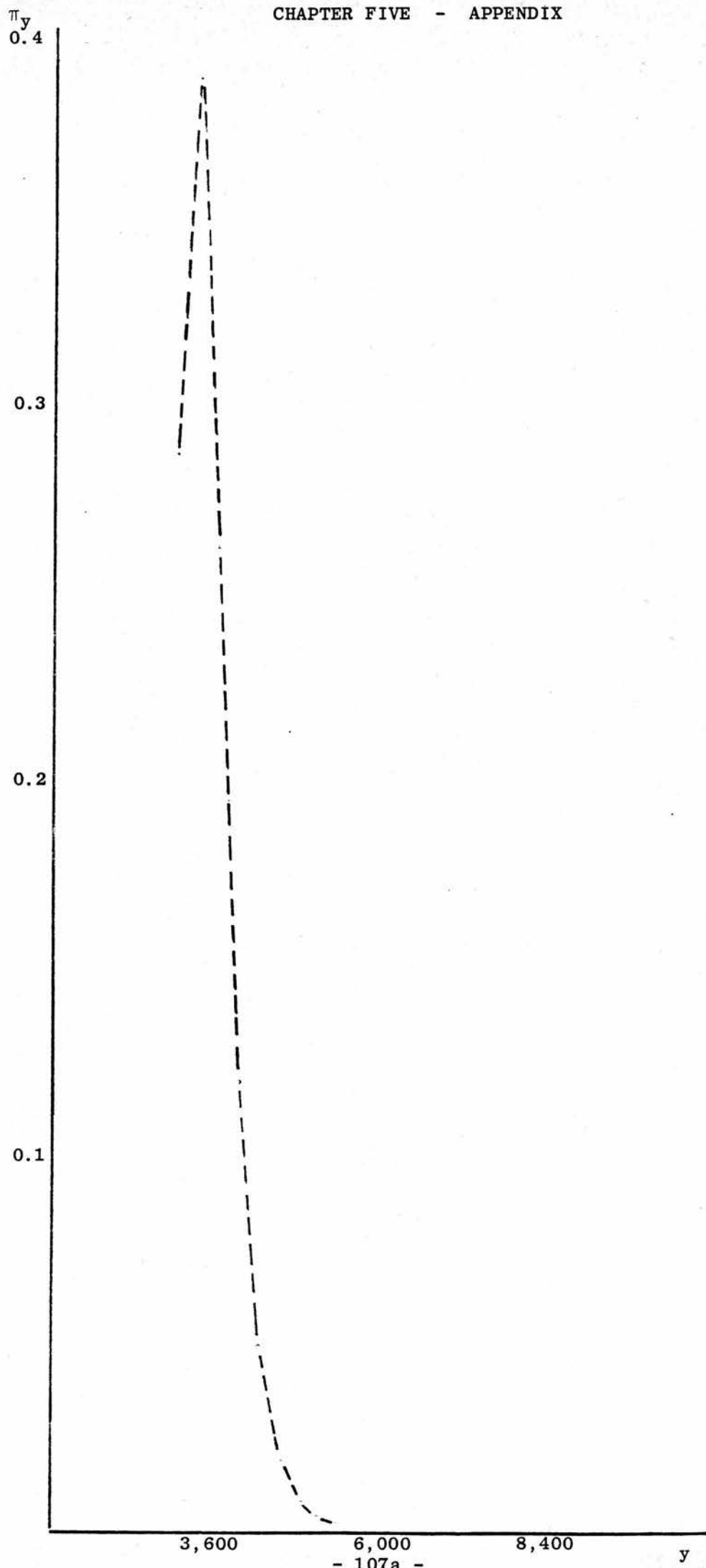
y	π_y
3000.00	0.2857141
3300.00	0.3853356
3600.00	0.2620399
3900.00	0.1200745
4200.00	0.0481578
4500.00	0.0191662
4800.00	0.0078952
5100.00	0.0034009
5400.00	0.0015319
5700.00	0.0007195
6000.00	0.0003510
6300.00	0.0001773
6600.00	0.0000925
6900.00	0.0000496
7200.00	0.0000274
7500.00	0.0000154
7800.00	0.0000089
8100.00	0.0000053
8400.00	0.0000032
8700.00	0.0000019
9000.00	0.0000012
9300.00	0.0000008
9600.00	0.0000005
9900.00	0.0000003
10200.00	0.0000002
10500.00	0.0000001
10800.00	0.0000001
11100.00	0.0000001
11400.00	0.0000000
11700.00	0.0000000
12000.00	0.0000000
12300.00	0.0000000
12600.00	0.0000000
12900.00	0.0000000
13200.00	0.0000000
13500.00	0.0000000
13800.00	0.0000000
14100.00	0.0000000
14400.00	0.0000000
14700.00	0.0000000
15000.00	0.0000000
15300.00	0.0000000
15600.00	0.0000000

Mean = 3,437.88

Variance = 162,350

Absolute skewness = 95,551,488

Relative skewness = 1.46



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CHAPTER 6: "EXTENSIONS: POSSIBLE EFFECTS OF AGE
AND ABILITY OR EDUCATION"

In our basic model, according to certain assumptions regarding the entry and exit of employees, we derived the distribution of earnings y for any specific wage structure within a firm or across firms; the probability mass function of the distribution was derived to be

$$\pi_y = \left\{ \frac{1}{a} \ln \left(\frac{y}{y_0} \right) - \sigma (bm)^0, \frac{1}{a} \ln \left(\frac{y}{y_0} \right) + 1 - \sigma (bm)^0 \right\}, \text{ where } \sigma(bm) \text{ was}$$

defined to be the Laplace transform of the probability density function of the promotion times for the product of the two parameters (b,m) , y_0 is the initial income and a is the constant income increment on a logarithmic scale. In this chapter we will examine the possible effects of age and ability or educational qualifications on the promotion process in addition to the fact that income promotion rates are functions of the time spent at each income level.

A. Age.

It is self-evident that physical age plays an important role in any theory seeking to explain the distribution of earnings not only because it is always present in the official earnings statistics (age-earnings profiles) but also because it is related to other factors directly influencing the marginal productivity of an employee. We quote from Klevmarken¹ *"The physical and mental ability of a young employee are usually higher than those of an old employee, and this should have an influence on marginal productivity and earnings in addition to experience. The rate of increase in earnings should therefore be a function, probably a decreasing one, of physical age"*. At this stage let us make another quotation from Lydall's multi-factor theory² of earnings distribution where he describes age *".... standing proxy for such factors as experience, learning by doing, and on-the-job training, and also for the effects of changes in ability, health and strength with age"*. Lydall does not discriminate between physical and active age (or length-of-service) but he clearly emphasizes the importance of the physical age factor.

Let us now examine two different models incorporating income increases due to age.

I) Income Increases Due to Age Occurring Outside the Wage Structure.

Let $Y(t)$ be a random variable representing the overall earnings of any employee at age t . The size of the earnings at that particular age can be attributed to two factors, physical age and the length-of-service, $Y(t)$ being their product. Therefore we assume that these two factors combine

multiplicatively. Quoting from Klevmarken *"The most important interaction (by statistical standards) is found between age and job level. The salary difference between high and low job levels increases by age or, to express it in another way, the salary differences between young and old employees are wider at high job levels than at low ones. One possible explanation for this result is that there is a wider scope for experience and skill at high job levels than at low levels"*.

Symbolically $Y(t) = x(t).y$, where ..(1)

y stands for the size of earnings according to our basic length-of-service model and $x(t)$ represents an income ladder where the growth of earnings is solely attributed to physical age. Let us consider the overall earnings difference of two individuals having the same earnings due to age but at different income levels in the length-of-service model, that is the case when $y_1 < y_2$ and $x_1(t) = x_2(t)$. The overall earnings difference will be: $Y_2(t) - Y_1(t) = x(t).(y_2 - y_1)$ and the greater the age the wider becomes the difference, whereas if the two factors combined additively the overall earnings difference would be independent of age.

Although it is more likely that age and the length-of-service are interdependent we will assume that they are independent and generated by two different stochastic processes. This condition appears to be rather stringent but it takes care of cases where external allocations occur and employees move from one firm to another in the same industry or across industries.

We have already examined and derived the probability mass function of earnings y due to the length-of-service. Let us now examine a stochastic model describing the influence of the age factor on the growth of earnings and

derive the probability mass function of the random variable $x(t)$. Since $Y(t)$ is the product of the two variables $x(t)$ and y , we will finally derive the overall distribution of earnings where both variables will be incorporated.

The Age Model.

A deterministic model could be a good description of the growth of individual earnings due to age since in many cases the earnings increases take place automatically. On the other hand, in seeking to describe reality we have to consider the unpredictable behaviour and change of policies of large corporations so that a probabilistic model appears to be the proper tool to represent the influence of the age factor upon earnings.

Let us assume that individual earnings grow according to a simple exponential equation, that is $x(t) = e^{dk}$, ..(2) where d is a constant income increment and k is a random variable representing the events or the times when the individual's earnings change as he ages.

We assume that $k = 0, 1, 2, \dots$ is generated by a non-homogeneous Poisson process, the intensity function of which decreases with age since the employee's physical and mental abilities or skills decline with age, and this is normally reflected on the growth of his earnings. Let $v(t)$ be the intensity function of the non-homogeneous Poisson process. The mean value^{*1} of the process is

*1 See Parzen.³

$m(t) = \int_{t_0}^t v(u) du$, where t_0 is the age at which the

individual becomes an active member of the labour force.*2

From (2) we have $k = \frac{1}{d} \ln x(t)$ and the probability mass function of the discrete random variable $x(t)$ is

$$\phi_t(x) = e^{-m(t)} \frac{\{m(t)\}^{\frac{1}{d} \ln x}}{(\frac{1}{d} \ln x)!} \quad \text{for } 1 < x < \infty. \quad \dots(3)$$

By definition when $t \leq t_0$ we have $k = 0$ and, consequently, $x(t) = 1$.

Finally, the derivation of the probability mass function of the product, $Y(t)$, of the two independent discrete random variables $x(t)$ and y , a well-known problem in the transformation of random variables theory, will proceed as follows:

Let $\pi\{Y(t)\}$ be the p.m.f. of $Y(t)$, then

$$\begin{aligned} \pi\{Y(t)\} &= \sum_{x=1}^{\infty} \phi_t(x) \cdot \pi_y = \frac{Y(t)}{x} = \\ &= \sum_{x=1}^{\infty} e^{-m(t)} \frac{\{m(t)\}^{\frac{1}{d} \ln x}}{(\frac{1}{d} \ln x)!} \quad \sigma \quad \frac{1}{a} \ln \left(\frac{Y}{x y_0} \right) \quad \{1 - \sigma(bm)\}. \end{aligned}$$

By making the substitution $i = \frac{1}{d} \ln x$, we obtain

*2

A specific intensity function that could be applied is of the following form: $v(u) = \delta e^{-\rho u}$, where δ and ρ are positive constants. The mean value function of the process will be:

$$m(t) = \int_{t_0}^t \delta e^{-\rho u} du = \frac{\delta}{\rho} (e^{-\rho t_0} - e^{-\rho t}).$$

$$\begin{aligned}
& \sum_{i=1}^{\infty} e^{-m(t)} \frac{\{m(t)\}^i}{i!} \frac{\frac{1}{a}(\ln Y - \ln y_0 - di)}{\sigma(bm)} \{1 - \sigma(bm)\} = \\
& = \frac{\frac{1}{a}(\ln Y - \ln y_0)}{\sigma(bm)} \{1 - \sigma(bm)\} e^{-m(t)} \sum_{i=0}^{\infty} \frac{\{m(t)\} \sigma^{\frac{d}{a}}(bm)}{i!} = \\
& = \frac{\frac{1}{a} \ln\left(\frac{Y}{y_0}\right)}{\sigma(bm)} \{1 - \sigma(bm)\} e^{m(t) \left\{ \sigma^{\frac{d}{a}}(bm) - 1 \right\}}, \quad \dots (4)
\end{aligned}$$

since the sum can be evaluated to give

$$e^{m(t) \sigma^{\frac{d}{a}}(bm)}$$

The above expression (4) is the probability mass function of the earnings of a homogeneous group of individuals having the same age and of any length-of-service. The final outcome is characterized by the existing similarity between the probability mass function of $Y(t)$ and that of y , derived as equation (9) in chapter five; the two expressions are identical except for the age-dependent term

$$e^{m(t) \left\{ \sigma^{\frac{d}{a}}(bm) - 1 \right\}}.$$

II) The Age Effect Incorporated into the Wage Structure.

Apparently we can influence the size of earnings if it is assumed that the income increment of our basic model is a function of age, that is $a = A(t)$. Since the length-of-service and age are assumed to be independent, the age t is treated as a parameter. Therefore, without any additional complications, the probability mass function of earnings y can be shown to be

$$\begin{aligned}
& \frac{1}{A(t)} \ln\left(\frac{y}{y_0}\right) \\
& \pi_{y(t)} = \{1 - \sigma(bm)\} \sigma(bm) \quad \dots (5)
\end{aligned}$$

As far as the promotion rates are concerned, the focal point of this chapter, we can incorporate the age effect into our basic model itself by assuming that the promotion rate is a function of the time spent in each income level and age.

Let T be a continuous random variable representing the time spent in each income level (promotion time). Any promotion rate which is a function of seniority within each income level can be defined in terms of the probability density function and the distribution of promotion times as follows:

$$\mu(\tau) = \frac{f(\tau)}{F(\tau)}, \text{ where } F(\tau) = \text{Probab. } \{T > \tau\}.$$

$\mu(\tau)d\tau$ is the conditional probability that the employee will be promoted to the next income level between τ and $\tau + d\tau$, provided that he has stayed in the previous one for time $T > \tau$. It can be shown that

$$F(\tau) = e^{-\int_0^\tau \mu(u)du} \quad \text{with initial condition } F(0) = 1.$$

It is most likely that the effect of age on the promotion rate is an additive one, although the possibility of a multiplicative effect should not be ignored. Let $\lambda(t)$ be the probability of promotion to the next income level that corresponds to the age effect; let also $\phi(\tau)$ be the probability of promotion that corresponds to seniority within each income level. The overall promotion rate will be

$$\mu_t(\tau) = \phi(\tau) + \lambda(t).$$

The corresponding distribution function of the promotion times will be

$$F_t(\tau) = e^{-\int_0^\tau \mu_t(u) du} = e^{-\lambda(t)\tau} \cdot e^{-\Phi(\tau)}, \text{ where}$$

$$\Phi(\tau) = \int_0^\tau \phi(u) du. \quad \text{Consequently the probability}$$

density function of the promotion times is

$$\begin{aligned} f_t(\tau) &= \phi(\tau) e^{-\Phi(\tau)} e^{-\lambda(t)\tau} + \lambda(t) e^{-\Phi(\tau)} e^{-\lambda(t)\tau} = \\ &= e^{-\lambda(t)\tau} \{ \phi(\tau) e^{-\Phi(\tau)} + \lambda(t) e^{-\Phi(\tau)} \}. \quad \dots(6) \end{aligned}$$

Following the same procedure as in our basic model let $\sigma_t(bm)$ be the Laplace transform of the p.d.f. $f_t(\tau)$ for bm , provided it exists. But $\sigma_t(bm) = \sigma_o\{bm + \lambda(t)\}$, where the latter is the Laplace transform of the expression $\phi(\tau)e^{-\Phi(\tau)} + \lambda(t) e^{-\Phi(\tau)}$.

Finally the probability mass function of earnings $y(t)$ will be

$$\pi_{y(t)} = [1 - \sigma_o\{bm + \lambda(t)\}] \sigma_o\{bm + \lambda(t)\}^{\frac{1}{a} \ln(\frac{y}{y_o})} \quad \dots(7)$$

B. Ability or Education.

Innate abilities, educational qualifications or attributes appraised by the employer are distributed among the firm's employees and they are usually taken into consideration by the firm's income policy makers. These characteristics may affect the wage structure in various ways. Let us consider a few obvious cases, but in order to facilitate the algebra we will assume that earnings grow linearly as the employee moves from one income level to the next, that is $y = y_0 + an$. Consequently, the probability mass function of earnings in our basic model will be:

$$\pi_y = \{1 - \sigma(bm)\} \sigma \left(\frac{1}{a} (y - y_0) \right) \quad \dots(8)$$

I Initial Income Corresponding to Different Levels of Ability or Education.

Suppose that in a particular firm an individual begins his career with an initial income y_0 which is now considered to be a random variable varying from employee to employee according to some innate ability or some acquired attribute (educational qualification).^{*3}

In our case let us assume that y_0 is a continuous random variable, independent of the length-of-service process, following a certain probability density function $g(y_0)$ for $c \leq y_0 < \infty$, where c is a minimum value. By the

*3

⁴ Fase in his econometric model of age-income profiles assumes that the initial income values follow the log-normal distribution.

randomization of the continuous parameter y_0 the probability mass function (8) becomes conditional and the unconditional mass function of earnings is the mixture of π_y and the probability density function $g(y_0)$. Therefore

$$\pi_y = \int_c^{\infty} \pi_y(y_0) g(y_0) dy_0.$$

In order to clarify the above randomization of the initial income values let us assume that $g(y_0)$ is a truncated negative exponential function with parameter ρ .

$$\begin{aligned} \text{Accordingly } \pi_y &= e^{\rho c} \int_c^{\infty} \{(1 - \sigma(bm))\} \sigma^{\frac{1}{a}(y - y_0)}(bm) \rho e^{-\rho y_0} dy_0 = \\ &= e^{\rho c} \rho \{1 - \sigma(bm)\} \sigma^{\frac{1}{a}y}(bm) \int_c^{\infty} \sigma^{-\frac{1}{a}y_0}(bm) e^{-\rho y_0} dy_0 = \\ &= e^{\rho c} \rho \{1 - \sigma(bm)\} \left\{ \frac{1}{a} \ln \sigma(bm) + \rho \right\} \left\{ \sigma^{-\frac{1}{a}y}(bm) e^{-\rho y} \right\}_c^{\frac{1}{a}y} \sigma(bm) \dots (9) \end{aligned}$$

II The Randomization of the Income Increment.

Productivity schemes or bonus schemes can be incorporated into an income promotion ladder by randomizing the income increment parameter a . Individuals whose performance has been highly evaluated by the employers are likely to receive higher remuneration than others in the event of income promotions. The differentiation of income increments can be even institutionalized as it often happens when educational qualifications are involved.

A simple model that could describe this case is the following:

let us assume that the income increment a is a discrete

random variable with an upper limit A. If we put $\frac{A}{a} = k$, and assume that $k = 1, 2, 3 \dots$ we have $a = \frac{A}{k}$, and substituting into (8), we obtain

$$\pi_y = \{1 - \sigma(bm)\} \sigma^{\frac{k}{A}(y - y_0)}(bm) \quad \dots(10)$$

This is the conditional probability mass function of earnings assuming the k has an observed value. Again in order to obtain the unconditional p.m.f. we have to multiply (10) by the probability mass function of k and sum-up over all possible values of k.

Suppose that k follows the geometric distribution with parameters p and q, then we have

$$\begin{aligned} \pi_y &= \sum_{k=1}^{\infty} \{1 - \sigma(bm)\} \sigma^{\frac{k}{A}(y - y_0)}(bm) p \cdot q^{k-1} = \\ &= p \{1 - \sigma(bm)\} \{ \sigma^{\frac{1}{A}(y - y_0)}(bm) - q \}^{-1} \quad \dots(11) \end{aligned}$$

III Income Promotion Rate Being a Function of the Length-of-Service.

In the last models of this section we will examine two possible effects of ability on the income promotion rates. First let us consider the case where the firm relates the ability of an employee to his length-of-service in the firm and, therefore, in the prevailing wage structure the income promotion rates become functions of the length-of-service variable. The point process in this case is a non-homogeneous Poisson process; in chapter five we have already examined the general non-Markovian case the promotion rates of which are functions of seniority within each income

level, and the specific Markovian case where the point process involved is a homogeneous Poisson one and the promotion rate is constant.

In our basic model we have established that the probability mass function of the number of promotions within a homogeneous group of employees with the same length-of-service is

$\pi_n(s) = P_n(s) - P_{n+1}(s)$, where \underline{n} corresponds to the number of income promotions, s represents the length-of-service variable and $P_n(s) = \text{Prob.}\{S_n \leq s\}$. The random variable $S_n = \tau_1 + \tau_2 + \dots + \tau_n$ represents the time required for the n th promotion to occur.

We have also derived the probability mass function of the number of promotions for a random group of employees irrespective of the length-of-service, it was found to be

$$\pi_n = \int_0^{\infty} \{P_n(s) - P_{n+1}(s)\} b m e^{-bms} ds, \text{ where } b m e^{-bms}$$

is the probability density function of the length-of-service in the firm.

The above integral can be evaluated in terms of the Laplace transforms to give

$$\pi_n = \Lambda_n(bm) - \Lambda_{n+1}(bm) \quad \dots(12)$$

where $\Lambda_n(bm)$ is the Laplace transform of the probability density function $p(S_n)$ for bm .

It has been shown^{*4} that the p.d.f. of the random variable S_n for the non-homogeneous Poisson process is

*4 See Parzen.⁵

$p(s) = e^{-m(s)} \frac{\{m(s)\}^{n-1}}{(n-1)!} v(s)$, where $v(s)$ is the intensity function and $m(s)$ the mean value function of the process respectively.

$$\text{Accordingly } \pi_n = \int_0^{\infty} e^{-m(s)} \frac{\{m(s)\}^{n-1}}{(n-1)!} v(s) e^{-bms} ds - \int_0^{\infty} e^{-m(s)} \frac{\{m(s)\}^n}{n!} v(s) e^{-bms} ds \quad \dots(13)$$

Although an explicit solution is not always obtainable, let us consider the case where the promotion rate grows exponentially with the length-of-service, that is $v(s) = \rho e^{\rho s}$ with $\rho > 0$. Then $m(s) = e^{\rho s}$ and (13) becomes

$$\pi_n = \int_0^{\infty} e^{-e^{\rho s}} \frac{(e^{\rho s})^{n-1}}{(n-1)!} \rho e^{\rho s} e^{-bms} ds - \int_0^{\infty} e^{-e^{\rho s}} \frac{(e^{\rho s})^n}{n!} \rho e^{\rho s} e^{-bms} ds.$$

By making the transformation $e^{\rho s} = x$ for $1 < x < \infty$, and with the Jacobian of the transformation being

$$|j| = \frac{1}{\rho} \frac{1}{x}, \text{ we obtain}$$

$$\pi_n = \int_1^{\infty} e^{-x} \frac{x^{n-1}}{(n-1)!} x^{\frac{-bm}{\rho}} dx - \int_1^{\infty} e^{-x} \frac{x^n}{n!} x^{\frac{-bm}{\rho}} dx.$$

Provided that $n > \frac{bm}{\rho}$ the approximate value of the two integrals is

$$\pi_n \approx \frac{bm}{\rho} \frac{(n - \frac{bm}{\rho} - 1)!}{n!} \quad \text{and by substituting } \frac{1}{a} (y - y_0) \text{ for}$$

n we derive the probability mass function of earnings

$$\pi_y \approx \frac{bm}{\rho} \frac{\{\frac{1}{a} (y - y_0) - \frac{bm}{\rho} - 1\}!}{\{\frac{1}{a} (y - y_0)\}!} \quad \dots(14)^{*5}$$

Finally we will examine a similar case by considering

*5

See Appendix at end of Chapter.

a point process, the promotion rate of which is a function of the number of income promotions that have already occurred. In order to simplify the presentation we will not follow our normal procedure but we will apply the results of the well-known Yule-Furry process.

First we will slightly alter our basic formulation regarding the growth of earnings and we will assume that earnings grow according to the following equation

$$y = y_0 + a(n-1) \text{ for } n = 1, 2, 3 \dots$$

We now assume that the promotion rate is defined by $\mu(\tau) = \lambda.n$, where λ is the chance of a given income promotion to initiate a new one in time $d\tau$, and n is the number of income changes already taken place in the interval $0 < S_n \leq s$.

Since the number of income changes follows the Yule-Furry process it can be shown that the probability mass function of \underline{n} in a homogeneous group of employees having the same length-of-service is

$$\pi_n(s) = e^{-\lambda s} (1 - e^{-\lambda s})^{n-1} \text{ with initial condition}$$

probab. $\{n(0) = 1\} = 1$. The p.m.f. of the number of promotions in a random sample of employees irrespective of length-of-service will be

$$\pi_n = \int_0^{\infty} e^{-\lambda s} (1 - e^{-\lambda s})^{n-1} b_m e^{-bms} ds, \text{ where } b_m e^{-bms}$$

is the probability density function of the length-of-service. If we consider the transformation $e^{-\lambda s} = x$, $0 < x < 1$, with the Jacobian being $|j| = \frac{1}{\lambda} x^{-1}$ the above integral can be evaluated to give

$$\pi_n = \frac{bm}{\lambda} \int_0^1 x^{\frac{bm}{\lambda}} (1-x)^{n-1} dx = \frac{bm}{\lambda} \frac{(\frac{bm}{\lambda})! (n-1)!}{(\frac{bm}{\lambda} + n)!} \quad \dots (14)$$

By substituting the earnings variable for \underline{n} we obtain the p.m.f. of earnings, that is

$$\pi_y = \frac{bm}{\lambda} \frac{(\frac{bm}{\lambda})! \{\frac{1}{a}(y - y_0)\}!}{\{\frac{bm}{\lambda} + \frac{1}{a}(y - y_0) + 1\}!} \quad \dots (15)^{*6}$$

IV Promotion Rates being Functions of Ability and Seniority within the Income Level.

In the previous model we examined the case where the firm evaluates an employee's abilities according to his length-of-service or the number of income promotions he has already achieved. Let us now examine one more extension of our basic model and assume that the promotion rate depends on the two factors, ability and seniority within the income level, which may combine additively; therefore the promotion rate $\mu_\lambda(\tau) = \phi(\tau) + \lambda$, where $\phi(\tau)$ is a function of seniority within the income level and λ the observed value of a characteristic which is distributed among the population of the firm's employees. On the additional assumption that λ follows a probability density function $g(\lambda)$ for $0 < \lambda < \infty$ the unconditional promotion rate can be obtained as follows: The conditional probability density function of the income promotion time is

$$f_\lambda(\tau) = \{\phi(\tau) + \lambda\} e^{-\int_0^\tau \{\phi(u) + \lambda\} du}, \text{ then the unconditional one}$$

$$\text{will be } f(\tau) = \int_0^\infty g(\lambda) f_\lambda(\tau) d\lambda = e^{-\int_0^\tau \phi(u) du} \int_0^\infty \{\phi(\tau) + \lambda\} e^{-\lambda\tau} g(\lambda) d\lambda$$

* 6

See Appendix at end of Chapter.

Similarly, the unconditional distribution function is

$$F(\tau) = \int_0^{\infty} g(\lambda) e^{-\int_0^{\tau} \{\phi(u) + \lambda\} du} d\lambda = e^{-\int_0^{\tau} \phi(u) du} \int_0^{\infty} g(\lambda) e^{-\tau \lambda} d\lambda$$

Therefore the unconditional promotion rate is

$$\mu(\tau) = \frac{\int_0^{\infty} \{\phi(\tau) + \lambda\} g(\lambda) e^{-\tau \lambda} d\lambda}{\int_0^{\infty} g(\lambda) e^{-\tau \lambda} d\lambda} = \phi(\tau) + \frac{\int_0^{\infty} \lambda g(\lambda) e^{-\tau \lambda} d\lambda}{\int_0^{\infty} g(\lambda) e^{-\tau \lambda} d\lambda}$$

We can proceed to the derivation of the probability mass function of earnings by means of the Laplace transform of the unconditional p.d.f. of the promotion times as in our basic model.

APPENDIX TO CHAPTER SIX.

A. The factorials appearing in the probability mass function of equation (14) can be evaluated by applying Stirling's formula

$$n! \cong \sqrt{2\pi} e^{-n} n^{n + \frac{1}{2}}.$$

Then the above p.m.f. takes the following form:

$$\pi_y = \frac{bm}{\rho} e^{\frac{bm}{\rho} + 1} \left[1 - \frac{\frac{bm}{\rho} + 1}{\frac{1}{a}(y - y_0)} \right]^{\frac{1}{a}(y - y_0) + \frac{1}{2}} \cdot \left[\frac{1}{a}(y - y_0) - \frac{bm}{\rho} - 1 \right]^{-\frac{bm}{\rho} - 1}$$

By changing the value of each one of the parameters, *ceteris paribus*, computer calculations have shown that relative shewness increases as either the rate of entry or the income increment increase. In addition, the initial income y_0 , the rate of exit and the parameter ρ of the non-homogeneous Poisson process are shown to be inversely related to relative shewness. Since the promotion probabilities depend on the total length of service by increasing the rate of exit, the movement of employees towards the higher income classes is reduced and, therefore, the inequality in the distribution decreases.

Table.

For a constant population of employees ($b=1$),
initial income $y_0 = 2,100$ £, constant income increment
 $a = 250$ £, rate of exit $m = 0.2$ and $\rho = 0.11$, the following
values for π_y have been obtained:

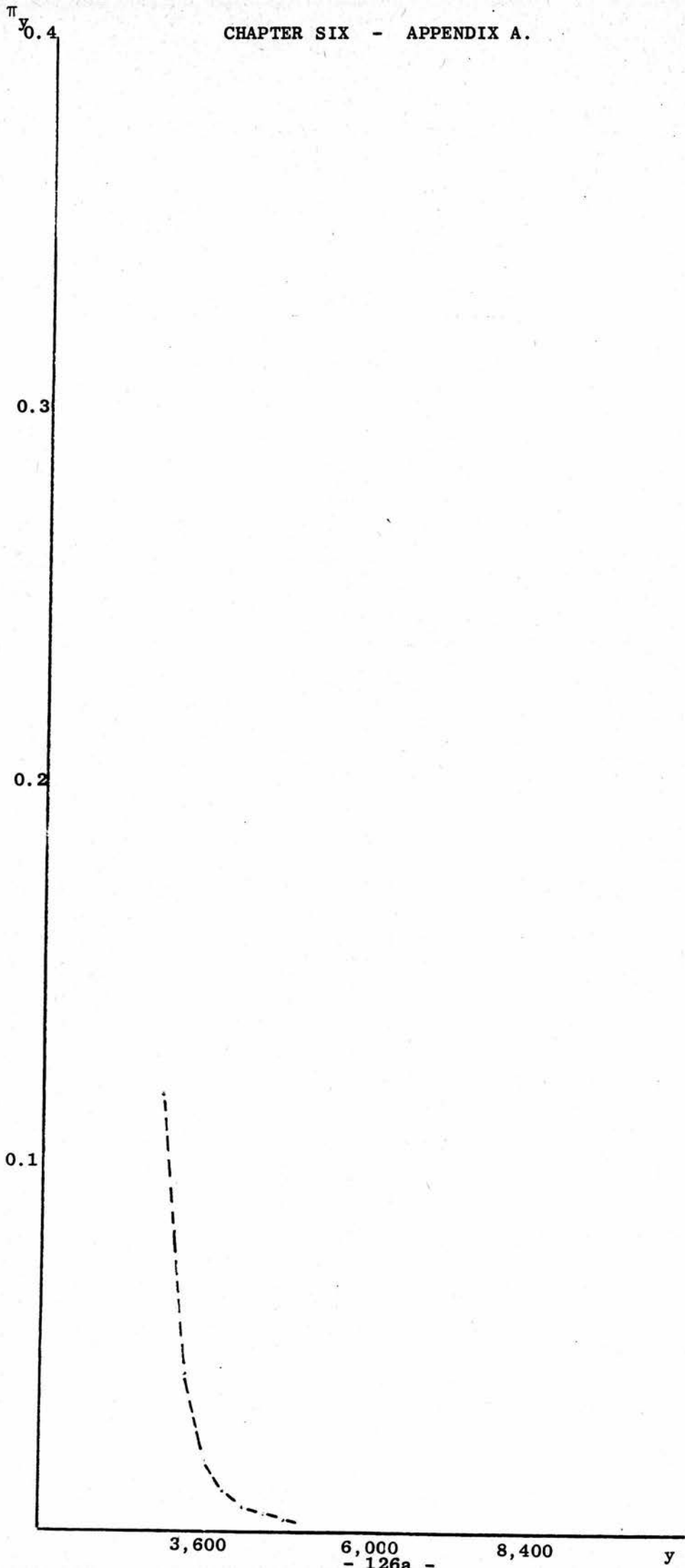
y	π_y
3000.	0.11633
3300.	0.04076
3600.	0.01889
3900.	0.01034
4200.	0.00630
4500.	0.00413
4800.	0.00287
5100.	0.00207
5400.	0.00155
5700.	0.00119
6000.	0.00094
6300.	0.00075
6600.	0.00061
6900.	0.00050
7200.	0.00042
7500.	0.00036
7800.	0.00030
8100.	0.00026
8400.	0.00023
8700.	0.00020
9000.	0.00017
9300.	0.00015
9600.	0.00014
9900.	0.00012
10200.	0.00011
10500.	0.00010
10800.	0.00009
11100.	0.00008
11400.	0.00007
11700.	0.00007
12000.	0.00006
12300.	0.00006
12600.	0.00005
12900.	0.00005
13200.	0.00004
13500.	0.00004
13800.	0.00004
14100.	0.00004
14400.	0.00003
14700.	0.00003
15000.	0.00003
15300.	0.00003
15600.	0.00003

Mean = 3,447.69

Relative skewness = 5.157

Variance = 1,037,047

Absolute skewness = 5,446,828,032



B. The p.m.f. of equation (15) is

$$\pi_y = \frac{bm}{\lambda} \frac{\left(\frac{bm}{\lambda}\right)! \left\{\frac{1}{a}(y - y_0)\right\}!}{\left\{\frac{bm}{\lambda} + \frac{1}{a}(y - y_0) + 1\right\}!}$$

Again, by applying Stirling's formula the factorials can be approximately evaluated so that the above function becomes

$$\pi_y = \left(\frac{bm}{\lambda}\right)^{\frac{bm}{\lambda} + 1.5} \cdot e^{\sqrt{2\pi}} \left[\frac{\frac{1}{a}(y - y_0)}{\frac{bm}{\lambda} + \frac{1}{a}(y - y_0) + 1} \right]^{\frac{1}{a}(y - y_0) + \frac{1}{2}} \cdot \left[\frac{bm}{\lambda} + \frac{1}{a}(y - y_0) + 1 \right]^{-\frac{bm}{\lambda} - 1}.$$

In this model our calculations have shown that relative skewness increases as either the rate of entry or exit increases. The promotion probabilities depend upon the number of previous income promotions, and by increasing the rates of entry or exit the expected number of income promotions for a period of completed length-of-service is reduced; the low income classes increase in size relative to the high income ones, and consequently, inequality increases.

On the other hand, relative skewness was shown to decrease when either the initial income y_0 , the parameter λ , or the income increment a increase. No apparent reason seems to justify the inverse relationship between the income increment a and relative skewness, although both the variance and absolute skewness were shown to increase with higher values of a .

Table.

Again for $b = 1$, rate of exit $m = 0.2$, initial income $y_0 = 2,700 \text{ £}$, constant income increment $a = 250 \text{ £}$ and $\lambda = 0.1$ we obtain the following values for the probability mass function:

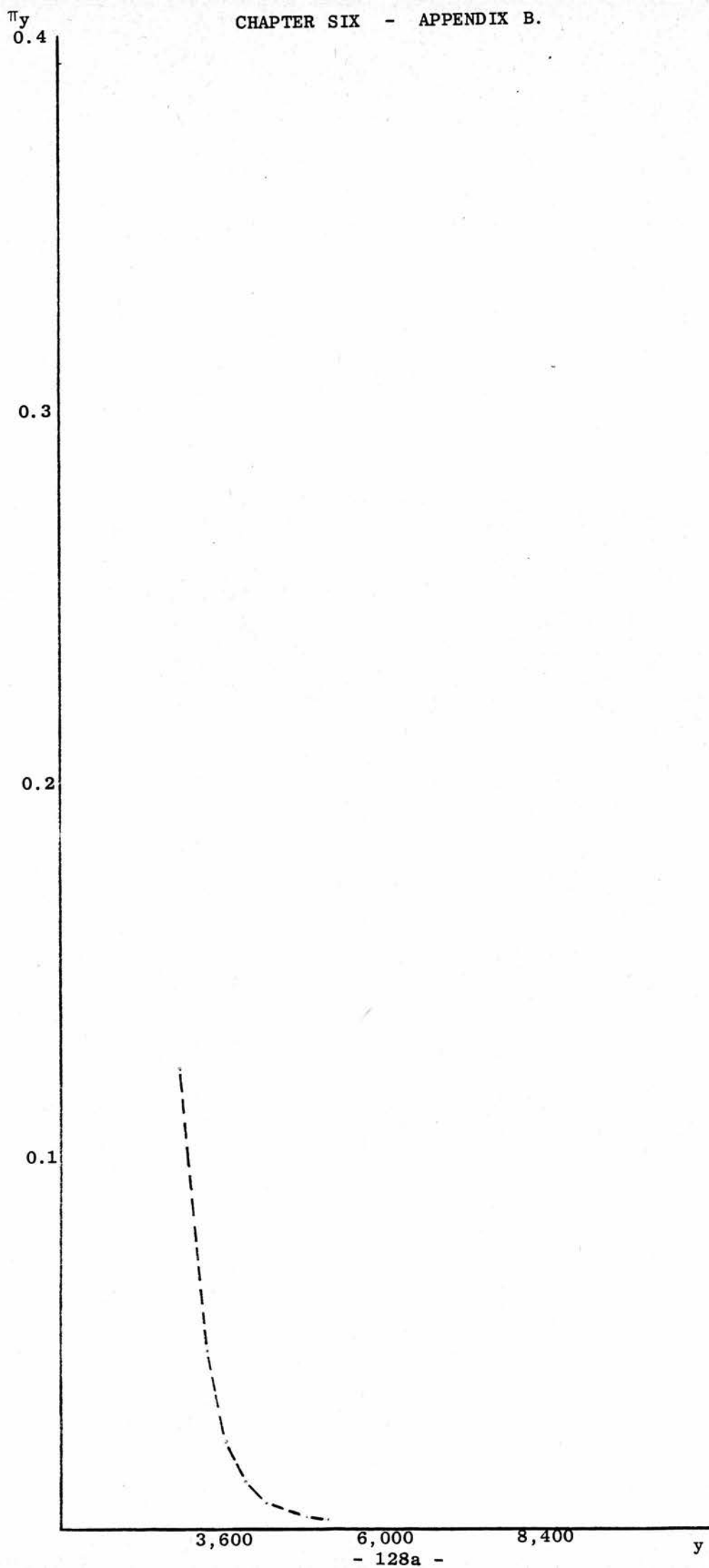
y	π_y
3000.	0.12369
3300.	0.04661
3600.	0.02234
3900.	0.01239
4200.	0.00758
4500.	0.00497
4800.	0.00343
5100.	0.00247
5400.	0.00184
5700.	0.00140
6000.	0.00110
6300.	0.00087
6600.	0.00071
6900.	0.00058
7200.	0.00048
7500.	0.00040
7800.	0.00034
8100.	0.00029
8400.	0.00025
8700.	0.00022
9000.	0.00019
9300.	0.00017
9600.	0.00015
9900.	0.00013
10200.	0.00012
10500.	0.00010
10800.	0.00009
11100.	0.00009
11400.	0.00008
11700.	0.00007
12000.	0.00006
12300.	0.00006
12600.	0.00005
12900.	0.00005
13200.	0.00005
13500.	0.00004
13800.	0.00004
14100.	0.00004
14400.	0.00003
14700.	0.00003
15000.	0.00003
15300.	0.00003
15600.	0.00002

Mean = 3,467.06

Variance = 1,023,154

Absolute skewness = 5,162,205,184

Relative skewness = 4.99



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 5. Parzen, E: op.cit., page 138.
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Chapter 7:

"EXTENSIONS: DEMOTIONS OR NEGATIVE INCOME CHANGES"

Within the framework of point processes we will seek to explain the distribution of the incomes of the self-employed and the earnings of the workers in the secondary market. Again we will be focussing our attention on the points of time when the income of a particular individual changes (event) rather than on the size of the income increment itself. The time parameter in our model will be the individual's age t - in this case the effect of the length-of-service is negligible - but, assuming that the individual enters the labour force at a certain age t_1 and that age is a simple linear transformation of the length-of-service, that is $t = s + t_1$, the time parameter of our point process can still be the length-of-service $s = t - t_1$ for $s > 0$.

A. The Earnings of Workers in the Secondary Market.

The secondary market is defined by Doeringer and Piore¹ as a group of low wage, marginal firms where work opportunities are casual and unstructured. Other characteristics are poor working conditions, little chance of promotion and on-the-job training, and personal (therefore arbitrary) administrative procedures. So far as the labour force in the secondary sector is concerned, workers have inadequate skills and educational qualifications, exhibit high turnover and their attitudes and demographic traits favour low job attachment. There is no complete and decisive dichotomy between the primary sector and the secondary one. Some jobs in internal labour markets with many entry ports, short mobility clusters and low paying work tend to resemble

the ones in the secondary sector. There is also some secondary employment attached to internal labour markets in which the remainder of the jobs are primary.

According to the segmented market theorists the factors that foster secondary labour markets are related to both the demand and the supply side of the market. Certain workers, particularly part-timers, due to demographic characteristics and social environment develop negative attitudes towards permanent employment and promotion opportunities. On the other hand, employers in the secondary sector cannot efficiently create internal labour market conditions and also, due to the technical aspects of certain jobs, they are indifferent to the reduction of labour turnover and to investment in training on the job. It is more likely that employee instability and unstable jobs are correlated, their joint effect being the establishment of secondary markets.

Whenever unstable employment relations exist, the costs of labour organization and bargaining power are high relative to the potential advantages obtained by the Trade-Union members. This is one of the reasons explaining the lack of unionization in the secondary market. Another, less obvious one, is the fact that many unions in the primary sector in order to guarantee permanent employment relations for their members are forced to adopt policies the side-effect of which is the existence of a labour pool of temporary workers. Therefore the costs of economic instability in the internal markets are conveyed to non-unionized internal markets and to the secondary sector.

Since in the secondary market there are no mobility clusters and institutionalized wage structures, the earnings

of a worker can be attributed to two independent factors, both of which are related to the individual's age. The first factor is the worker's physical strength or abilities. With increasing age, we can assume that his earnings begin to decrease as soon as he enters the labour force, and assuming an exponential decline of earnings, this factor can be defined as $e^{-a k_1 s}$, where a is a constant increment on a logarithmic scale, and k_1 is a random variable generated by a point (Poisson) process. The point events of the process correspond to the negative changes of the worker's earnings. The second factor is discipline or the ability to conform. This ability increases with age, and so it contributes to the worker's growth of earnings. Again we assume an exponential growth of earnings, that is the second factor can be defined as $e^{a k_2 s}$, where a is, as above, the constant increment and k_2 is a random variable generated by a Poisson process. This time the point events of the process correspond to positive changes of the worker's earnings.

If we assume that the above factors combine multiplicatively and that each worker starts with an initial income value X_0 , then his earnings at age $s = t - t_1$ will be:

$$y(s) = X_0 e^{a\{k_2(s) - k_1(s)\}} \quad \dots(1)$$

When $t = t_1$ (or $s = 0$) by definition we have $k_2, k_1 = 0$.

Another case may arise when we have a linear decline or growth of earnings and the two factors affect the individual's earnings additively, so that the worker's earnings can be $y(s) = X_0 + a\{k_2(s) - k_1(s)\}$. Our analysis will be concerned with the former case.

Before we proceed with the mathematical formulation of the model, we would like to elaborate on one particular aspect of the worker's retirement. As long as the worker

is an active member of the labour force we will always have $k_2 \geq k_1$, since it is very unlikely for a worker to accept a remuneration lower than his initial income, that is the income he started with at the minimum age he was qualified to enter the labour market. Due to government intervention (establishment of minimum rates) we will assume that if the worker's earnings fall below X_0 , then the worker is automatically considered to be retired. Therefore $k_2 - k_1 < 0$ is an absorbing state for the process.

NB: Given that the point processes are of the Poisson type, and if we assume that not all the events corresponding to income changes are counted but each positive change occurring is counted with a probability p and each negative change is counted with a probability q , then the point processes are still of the Poisson type.²

I. The Model (Randomized Random Walks).

In order to derive the distribution of earnings of a manual worker, we have to examine the probability mass function of the discrete random variable $k_2 - k_1$, in essence the r.v. $k_2 - k_1$ represents the difference between two Poisson variables. Let us assume that the expectation of the process k_2 is λs and the expectation of the process k_1 is ρs . It has been shown by W.Feller³ that the distribution of the difference of two independent Poisson variables is identical with the distribution generated by a time-dependent randomized random walk with the probability of one step upwards (or positive jump) being $\frac{\lambda}{\lambda + \rho}$, and the probability of a negative jump being $\frac{\rho}{\lambda + \rho}$. We also have to note that in the above-mentioned random walk the time between two

consecutive transitions follows the negative exponential distribution with expectation $\{(\lambda + \rho)s\}^{-1}$.

The probability mass function of the random variable $k = k_2 - k_1$ is

$$P_S(k = k_2 - k_1) = \left(\frac{\lambda}{\rho}\right)^{\frac{k}{2}} e^{-(\lambda + \rho)s} \mathbb{I}_k(2s\sqrt{\lambda \cdot \rho}). \quad (2)$$

for $k = 0, \pm 1, \pm 2, \pm 3, \dots$

\mathbb{I}_k is known in the literature as the Bessel function of imaginary argument. The above function of order k is defined by:

$$\mathbb{I}_k(2s\sqrt{\lambda \cdot \rho}) = \sum_{v=0}^{\infty} \frac{(s\sqrt{\lambda \cdot \rho})^{2v+k}}{v!(k+v)!}, \text{ and it has a generating function of the form}$$

$$\sum_{k=-\infty}^{+\infty} n \mathbb{I}_k(x) = e^{\frac{1}{2}x(u+u^{-1})}$$

From equations (1) and (2) we can derive the probability mass function of the earnings y ,

$$P_S(y) = \left(\frac{\lambda}{\rho}\right)^{\frac{1}{2a} \ln(\frac{y}{X_0})} e^{-(\lambda + \rho)s} \mathbb{I}_{\frac{1}{a} \ln(\frac{y}{X_0})}(2s\sqrt{\lambda \cdot \rho}), \quad (3)$$

for $X_0 \leq y < \infty$, since $y < X_0$ is an absorbing state. The probabilities $P_S(y)$ correspond to the distribution of earnings of a sample of secondary workers in the same age.

II The Distribution of Earnings of a Mixed Sample of Secondary Workers.

If we consider a random (mixed) sample of individuals heterogeneous in age, we will have to treat age as a continuous random variable and introduce the limiting age distribution of the secondary workers. Finally, we will have to integrate over all possible cohorts of workers with different

ages, thus obtaining the probability of any worker, regardless of age, having earnings y . We can apply the assumptions and procedure we followed in order to derive the limiting length-of-service distribution in the case of primary employees, but we have to elaborate on the assumptions related to the rate of exit of employees in the secondary sector. First, we assume that due to demographic reasons there is a constant rate of exit d . Secondly, we have to take into account the absorbing state of the process, that is the probability that the worker's earnings will decline and become less than his initial income value, so that he will be forced to leave.

Let $u(s)^{*1} = \text{Probab.}\{y(s) \geq X_0\}$, the overall exit rate will be $\frac{u'(s)}{u(s)} + d = h(s)$ for $s > 0$, and the survival probability, as an active member in the labour force, will be:

$\gamma(s) = u(s) e^{-ds}$ for $s > 0$, with boundary condition $\gamma(0) = 1$.

Again we assume that there is a constant rate of entry per worker b , and the Malthusian parameter is the root $\underline{\varepsilon}$ of the equation $\phi(\varepsilon) = b \int_0^{\infty} e^{-\varepsilon s} d\gamma(s) = 1$. Since in this case the parameter $\underline{\varepsilon}$ cannot be easily estimated in terms of the entry and exit rates, the limiting age distribution will be a function of it. As in chapter five $\omega(T)$, being the expected number of individuals who enter the secondary market in a small interval near T , asymptotically tends to $No b e^{\varepsilon T}$. Therefore the number of workers of age $s = t - t_1$ years at time T approaches, for $T - s$ sufficiently large,

$$v(s, T) \rightarrow No b e^{\varepsilon(T-s)} \gamma(s).$$

*1 In essence $1 - u(s)$ is the probability of the first passage through $k = k_2 - k_1 < 0$ at epoch s . See W. Feller⁴, and D. R. Cox and H. D. Miller⁵.

The limiting age distribution is

$$\frac{\int_0^{\infty} v(s, T) ds}{\int_0^{\infty} v(s, T) ds} = \frac{\int_0^{\infty} \text{Nob } e^{\varepsilon(T-s)} \gamma(s) ds}{\int_0^{\infty} \text{Nob } e^{\varepsilon(T-s)} \gamma(s) ds} = \frac{\int_0^{\infty} e^{-\varepsilon s} \gamma(s) ds}{\int_0^{\infty} e^{-\varepsilon s} \gamma(s) ds}$$

$$\text{But } \int_0^{\infty} e^{-\varepsilon s} \gamma(s) ds = \frac{1 - \int_0^{\infty} e^{-\varepsilon s} d\gamma(s)}{\varepsilon} = \frac{1 - \frac{1}{b}}{\varepsilon} = \frac{b-1}{b\varepsilon}.$$

Finally we obtain

$$\frac{\int_0^{\infty} v(s, T) ds}{\int_0^{\infty} v(s, T) ds} = \frac{\varepsilon b e^{-\varepsilon s} \gamma(s)}{b-1} = \frac{\varepsilon b}{b-1} e^{-(\varepsilon + d)s} u(s). \quad (4)$$

By combining (3) and (4) and integrating we obtain

$$\pi_y = \int_0^{\infty} \frac{P_s(y)}{u(s)} \frac{b\varepsilon}{b-1} e^{-(\varepsilon + d)s} u(s) ds, \text{ in the distribution}$$

we are counting the workers who are still in the labour force and, consequently, we have to normalize $P_s(y)$ by the factor $\frac{1}{u(s)}$.^{*2}

The integral $\int_0^{\infty} \frac{b\varepsilon}{b-1} e^{-(\varepsilon + d)s} P_s(y) ds$ is, in essence the Laplace transform of $P_s(y)$ for $\varepsilon + d$ and it can be evaluated to give⁷

$$\pi_y = \frac{b\varepsilon}{b-1} \frac{\delta^{-\frac{1}{a} \ln(\frac{y}{X_0})}}{\lambda(\delta - \xi)}, \text{ where} \quad (5)$$

$$\delta = \frac{\lambda + \rho + \varepsilon + d + \sqrt{(\lambda + \rho + \varepsilon + d)^2 - 4\lambda\rho}}{2\lambda} \text{ and}$$

$$\xi = \frac{\lambda + \rho + \varepsilon + d - \sqrt{(\lambda + \rho + \varepsilon + d)^2 - 4\lambda\rho}}{2\lambda}$$

Provided that $\rho + \varepsilon + d - \lambda > 0$ we will have $\delta > 1$ and by putting $\ln \delta = c$ and substituting into (5) we obtain

^{*2} For a similar treatment see J. Steindl.⁶

$\pi_y = \frac{b\varepsilon}{b-1} \frac{1}{\lambda(e^c - \xi)} \left(\frac{x_0}{y}\right)^{\frac{c}{a}}$, this being the Pareto law
 in the discrete form. The Pareto coefficient $\frac{c}{a} = \frac{\ln \delta}{a}$
 depends upon the rates of the two Poisson processes, the
 income increment on a logarithmic scale, the Malthusian
 parameter ε and upon the demographic rate of exit d .

As in our basic model our aim has not been to derive
 the Pareto distribution but, rather, to analyze the mechan-
 ism of the distribution of earnings in the secondary market
 applying the theory of point processes. Our basic assumption
 in this model has been that in the secondary market positive
 or negative income increments occur at random due to unstable
 employment relations. The application of Non-Markovian point
 processes, where the hazard functions depend upon the
 worker's age, could be a more realistic approach but it would
 greatly complicate the derivation of explicit mathematical
 results.

B. The Incomes of the Self-employed.

As in the model explaining the earnings of secondary workers we must allow for demotions or negative income increments. The income of a self-employed fluctuates in the same way as the sales of a firm increase or decrease at random points in time.

It would be more realistic to assume that in the case of the self-employed the size of the income increment is not a parameter determined by government legislation or customary law, it is a random variable that could be related to the number of positive income changes in the sense that as the self-employed person gains in reputation he demands a greater reward for his services. But this is not always the case since the individual's reward (or fees) may be constrained by institutional regulations or professional ethics (according to the specific service or commodity provided). Apart from the above constraint the person might obtain monopoly power (i.e. a famous surgeon and, consequently, he will charge discriminating fees to different customers in order to maximize his profits). Therefore, generally speaking, we will assume that the size of the income increment is a random variable following a distribution independent of the point process.

The Model.

Let again k_2 be a random variable corresponding to the number of positive income changes (events) and k_1 be a random variable corresponding to the number of negative income changes. Also let a be a random variable, independent of the point process, representing the size of the income increment.

The random variables k_2 and k_1 are again generated by two independent Poisson processes with expectations λs and ρs respectively, λ and ρ being positive constants. The independence assumption behind the two processes appears to be a realistic one. The loss of a customer that could be associated with a negative event has no effect upon the individual's probability of acquiring a new customer (positive event) unless we assume that in the market exists perfect knowledge on the part of all customers (or buyers), the latter case representing a completely idealised situation.

Assuming that the person's income grows exponentially his income level at age $t = s + t_1$ will be

$$y(s) = e^{a\{k_2(s) - k_1(s)\}} - 1 \quad (6)$$

for $0 \leq y(s) < \infty$, that is $k_2 - k_1 < 0$ is an absorbing state for the process. When $t = t_1$ or $s = 0$ we have by definition $k_2 = k_1 = 0$ or $y(s) = 0$.

If we follow the same procedure as in the case of secondary workers, and provided that the random variable \underline{a} has an observed value \hat{a} , the conditional probability mass function of \underline{y} will be:

$$\pi_{y/a=\hat{a}} = \frac{b\varepsilon}{b-1} \frac{1}{\lambda(e^c - \xi)} \left(\frac{1}{y+1}\right)^{\frac{c}{\hat{a}}}. \quad (7)$$

Let us now assume that \underline{a} is a discrete random variable with an upper limit A . If we put $\frac{A}{a} = x$, for $x = 1, 2, 3, \dots$, we have $\frac{1}{a} = \frac{x}{A}$, and substituting into (7) we obtain

$$\pi_{y/x=\hat{x}} = \frac{b\varepsilon}{b-1} \frac{1}{\lambda(e^c - \xi)} \left(\frac{1}{y+1}\right)^{\frac{c\hat{x}}{A}}. \quad (8)$$

In order to obtain the unconditional probability mass function of incomes we have to multiply (8) by the p.m.f. of \underline{x} and sum up over all possible values of \underline{x} .

Suppose that \underline{x} follows the geometric distribution with parameters \underline{p} and \underline{q} , then we have

$$\begin{aligned}\pi_y &= \sum_{x=1}^{\infty} \frac{b\varepsilon}{b-1} \frac{1}{\lambda(e^c - \xi)} \left\{ (y+1)^{-\frac{c}{A}} \right\}^x p q^{x-1} = \\ &= \frac{p b \varepsilon}{(b-1) \lambda (e^c - \xi)} \frac{1}{(y+1)^{\frac{c}{A}} - q} \quad \dots (9)\end{aligned}$$

We should note, however, that a more realistic approach would include the assumption of a minimum critical income value, different from the institutionalized initial level of earnings assumed in the case of secondary market workers. This minimum income, y_{\min} , may be related to minimum subsistence level and to fixed costs that a self-employed person has to face.

Without altering the structure of the model the final p.m.f. of incomes becomes in this case:

$$\pi_y = \frac{p d \varepsilon}{(b-1) \lambda (e^c - \xi)} \left\{ \left(\frac{y}{y_{\min}} \right)^{\frac{c}{A}} - q \right\}^{-1} \quad \dots (10)$$

Since $q < 1$ and for large values of y the above probability mass function, just like the one in (9), can be approximated by the Pareto law.

As far as inequality is concerned one important conclusion that can be derived is that an increase in the income increment, in the case of secondary market workers, or an increase in the maximum income value A , in the case of self-employed individuals, decreases the Pareto coefficient and, consequently, increases the inequality of incomes.

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CHAPTER 8: REPERCUSSIONS, AND A CONCLUDING NOTE.

A. Repercussions Related to Consumption Theory.

An important repercussion of our proposed models may be related to the application of the theory of consumer behaviour in internal labour markets and, in particular, to the specification of the relationship between labour incomes and individual consumption in the above markets. There are, however, other variables which influence the level of individual consumption; socio-economic characteristics such as age, family size, wealth and expectations play a vital role in determining the amount of spending of an employee.

In the literature there are four main theories postulating a particular relationship between consumption and income in terms of individual behaviour, and at a later stage applied to aggregate behaviour. Before we put forward a hypothesis elucidating the effect of earnings upon individual consumption in internal markets let us examine each general theory in turn.

I The Absolute Income Hypothesis.

This hypothesis originated from Keynes¹ and its two essential points are that real consumption expenditures are a stable function of real income^{*1} and that the marginal propensity to consume is a positive number, less than one. The hypothesis has followed one of two forms; the first form relates the level of consumption to current income and other variables, that is $C = a + bY + cZ + u$, where C represents consumption, Y is income, Z is a set of other variables, u is a stochastic term and a, b, c are parameters. The second form is $\frac{C}{Y} = a_1 + b_1Y + c_1Z + u_1$, where $\frac{C}{Y}$ is the consumption to income ratio. In the above two relations u or u_1 are assumed to be independent of consumption, an assumption that Ferber³ considers rather unrealistic^{*2}.

*1 In fact Keynes supported the thesis that consumers were not subject to money illusions but, as Ackley² points out, this may not be true at least in the short run.

*2 We must note that in theory consumption functions are the complement of saving functions and they can be applied interchangeably in the hypothesis.

II The Relative Income Hypothesis.

Brady and R.Friedman⁴ introduced the assumption that the saving rate depends on the relative position of the individual on the income scale and not on the absolute level of income. Therefore, incorporating the above assumption into the consumption equation, we obtain

$\frac{C}{\hat{Y}} = a + b \frac{Y}{\hat{Y}}$, where a , b are constants and \hat{Y} represents the average income. At a later stage Duesenberry⁵ provided psychological support - within a social framework - for this hypothesis by arguing that individuals have a strong tendency to emulate their neighbours and also to strive towards a higher standard of living. His main points were that an individual's consumption habits are influenced by the consumption habits of those with whom he associates, and that it is harder for a family to reduce expenditures than to refrain from making them in the first place. For this reason people seek to maintain at least the highest standard of living obtained in the past, and once a new higher standard of living is attained, individuals are reluctant to return to a lower level when their incomes go down.

According to Duesenberry, *ceteris paribus*, a person's average propensity to save will be a rising function of his percentage position in the income distribution. More specifically, a person's utility function depends on relative consumption expenditures, that is

$$U_i = f\left(\frac{C_{i1}}{R_i}, \frac{C_{i2}}{R_i}, \dots, \frac{C_{in}}{R_i}, \frac{A_{i1}}{R_i}, \dots, \frac{A_{in}}{R_i}\right), \text{ where } U_i \text{ is}$$

the utility function of the i th individual, A_{ik} the value of assets of the i th individual in the k th period, C_{ik} is the consumption expenditure of that person in the k th period and $R_i = \sum_j a_{ij} C_j$ is the weighted average of the consumption expenditure of others in the community. Each person maximizes his utility function subject to current and future incomes, interest rates and assets.

Duesenberry inferred that from a time-series point of view the relative income hypothesis can be expressed by making the consumption rate a function of the ratio of current income to the highest level of income previously obtained, a simple algebraic form being

$$\frac{C_t}{Y_t} = a + b \frac{Y_t}{Y_0}, \text{ where } Y_0 \text{ stands for previous peak income value.}$$

Davis⁶ suggested a different version of the relative income hypothesis by substituting previous peak consumption for previous peak income. He argued that people become adjusted to a certain level of consumption rather than to a level of income and, consequently, it is past spending that influences current consumption.

Davis based his version of the hypothesis on the additional argument that current income is likely to be less stable and less representative of a family's living standard than current consumption.

III The Permanent Income Hypothesis.

In an attempt to explain the stability of consumption outlays among non-wage earner families with varying incomes from period-to-period, M. Friedman⁷ put forward his permanent income hypothesis which is based upon three assumptions. First a consumer unit's observed income Y and consumption C in a particular period consist of two components, the "transitory" and the "permanent" one.

$$\text{Symbolically } Y = Y_p + Y_t$$

$$C = C_p + C_t.$$

The permanent income is the average income a household expects to earn during its planning horizon; in a given period of time it is a function of the wealth of the consumer unit, estimated as the discounted present value of a stream of future expected receipts, and the rate \underline{r} (or a weighted average of a set of rates) at which the expected receipts are discounted.

The second assumption is that permanent consumption is a multiple \underline{k} of permanent income, that is

$C_p = k(i, w, u) \cdot Y_p$, where k , being a factor of propensity, is a function of the interest rate i , the ratio of non-human to total wealth w , and a variable u of which age and tastes are principal components. Therefore Friedman assumes that k and Y_p are independent.

Thirdly, the transitory and permanent income components are assumed to be uncorrelated, the same assumption being applied to transitory and permanent consumption, and to transitory consumption and transitory income. Friedman's example of the permanent income hypothesis is the following: Let R_1, R_2 be an individual's income in years 1 and 2, and

let i be the rate of interest. The individual's expected wealth at the beginning of year 1 is

$w_1 = R_1 + \frac{R_2}{1+i}$, and so the consumption expenditure

in year 1 is $C_1 = f(w_1, i) = f(R_1 + \frac{R_2}{1+i})$.

More generally $C_{pt} = f(Y_{pt}, i) = F(w_t, i)$, where t refers to a specific point in time.

Friedman argues that at a certain point on the income-axis mean measured income, \hat{Y}_m , is equal to mean permanent income, \hat{Y}_p . Individuals who have a permanent income greater than the mean one tend to have a positive transitory income too. On the other hand, those who have $\hat{Y}_p < \hat{Y}_m$ tend to have a negative transitory income.

Summing up, according to this hypothesis the consumption outlays are a constant proportion k of the permanent income level^{*3}, the value of k varying for consumer units of different types and of different tastes. The observed level of consumption and income deviate from the permanent (or planned) levels by the amount of the transitory components which are random and independent of each other.

There is considerable doubt regarding the validity of two assumptions, the independence between the proportionality factor k and the level of income, and the lack of correlation between transitory consumption and transitory income. Friend and Kravis⁸ remark that according to the permanent income hypothesis "*low-income families will have no*

*3

Friedman suggests that a proper weighted average of the incomes of a homogeneous group (i.e. occupation) seems to be a good approximation for the permanent components for that group.

greater preference for purchase of future goods than will high-income families". Duesenberry⁹ raises the same point.

The basic argument is that consumer units at different income levels will have different kinds of pressures and motivations. The assumption of a zero marginal propensity to consume out of transitory income is also questionable since low-income consumer units are under strong pressures to spend any unexpected income to meet current needs, and since an unequal distribution of wealth can persuade low-income families to save in order to maintain consumption in the face of temporary declines in income.

Hamburger¹⁰ suggested a different version of the permanent income hypothesis by postulating that the total current consumption outlays depend mainly on tastes, the interest rate and the discounted value of life-time resources which are in turn determined by the sum of wealth and a multiple of the consumer's current wage rate.

IV The Life-cycle Hypothesis.

Modigliani, Ando and Brumberg¹¹ gave a different interpretation to the concept of permanence by considering permanent wealth rather than permanent income. They assumed that the utility function of an individual depends upon his aggregate consumption in current and future periods until death. The consumer maximizes his utility function subject to the available resources, the resources being the sum of current and discounted future earnings over his life-time and his current assets. The amount allocated to consumption is a certain proportion of these resources, that is the utility function is homogeneous - of any positive degree - with respect to consumption at different points in time.

The planning horizon of the individual is his whole life-time. Persons try to spread life-time consumable resources evenly over their lives. More specifically they accumulate enough savings during their earning years to maintain the same standard of living during retirement. The division of life between the earning-span and retirement is taken as an institutional fact.

Finally, it is assumed that the individual neither expects to receive nor desires to leave any bequest; the proponents of the hypothesis, however, do not consider the above assumption a very stringent one.

The hypothesis can be expressed as follows:

$C_t^T = k_t^T U_t^T$, where, as in the permanent income hypothesis, k_t^T is a factor of proportionality which depends upon the form of the utility function (the consumer's tastes), the interest rate and the present age of the person. C_t^T is total consumption of a person of age T in year t . One of the most interest-

ing aspects of the hypothesis is the factor U_t^T , the present value of resources at age T and in year t. The above factor can be defined by the following equation:

$$U_t^T = a_{t-1}^T + Y_t^T + \sum_{\tau=T+1}^n \frac{Y_t^{T\tau}}{(1+i_t)^{\tau-T}}, \text{ where } Y_t^T \text{ denotes}$$

non-property income at age T and in year t, and i_t is the real rate of return on assets. The variable $Y_t^{T\tau}$ represents the expected earnings of an individual T in the τ th year of his life.

$$\text{The component } \sum_{\tau=T+1}^n \frac{Y_t^{T\tau}}{(1+i_t)^{\tau-T}} = \hat{Y}_t^T (n-T), \text{ where } \hat{Y}_t^T \text{ is}$$

the average annual expected income and n is the earning-span of the individual.

Finally, according to the life-cycle hypothesis, the total consumption of an individual of age T in the year t will be:

$$C_t^T = k_t^T Y_t^T + k_t^T (n-T) \hat{Y}_t^T + k_t^T a_{t-1}^T.$$

The next step considered by Aldo and Modigliani is the aggregation procedure, first they aggregate within each age group and then over all the age groups. In case k_t^T is different for each individual in the age group the authors argue that under certain conditions, stated by Theil,¹² k_t^T can be shown to represent the weighted averages of the individual proportionality factors.

V On Consumption Theory in Internal Labour Markets:

A Proposed Synthesis.

By presenting the above brief survey of the four main hypotheses regarding consumer behaviour our intention has not been to emphasize the short-comings of each theory, this would require a detailed critical analysis of both the theoretical and the empirical^{*4} part of the theories. On a micro-economic basis each hypothesis suggests a certain relation between consumption and income; the latter is considered to be a general concept including earnings and income from other sources, although the life-cycle hypothesis proceeds a step further and considers the earnings of an individual separately. This is a point that has not been given considerable attention by the hypotheses, that is the lack of any insight into the relation between individual consumption and earnings.

We consider this problem even more important in internal labour markets with their stable employment relations and well-defined income ladders which determine the employee's labour income streams across his earning-span and, consequently affect his consumption pattern.

The proposed relation between earnings and an employee's consumption pattern in the internal market is rather a synthesis than a rejection of the assumptions incorporated in the four hypotheses.

Apart from other variables that are not considered in this model, the total consumption of an individual i with

^{*4} Numerous tests of each hypothesis exist in the literature; the most recent and integrated one is by Singh, Drost and Kumar.¹³

s years of service in the firm j, can be a function of his current earnings, the expected value of the earnings of employees with the same length-of-service and the expected value of the earnings of all employees employed by firm j. Symbolically we have $C_{ijs} = f(Y_{ij}(s), \hat{Y}_j(s), \hat{Y}_j)$, where C_{ijs} is the consumption of the individual i in firm j, and with s years of service. $Y_{ij}(s)$ corresponds to the earnings of the employee i in the firm j with s years length-of-service; the dependence of consumption on this variable is in accordance with the Keynesian hypothesis that current consumption depends on current income.*⁵

The second variable $\hat{Y}_j(s)$ is the expected value of the probability mass function

$$\pi_y(s) = P_{\frac{1}{a}} \ln \left(\frac{y}{y_0} \right) (s) - P_{\frac{1}{a}} \ln \left(\frac{y}{y_0} \right) + 1 (s), \text{ derived}$$

as equation (2) in chapter five. It is reminiscent of Aldo and Modigliani's concept of "average expected income", and in our case it corresponds to the average earnings the employee expects to earn at the sth year of his length-of-service in firm j.

Finally the third variable, \hat{Y}_j is the expected value of the probability mass function

$$\pi_y = \sigma \left(\frac{1}{a} \ln \left(\frac{y}{y_0} \right) \right) - \sigma \left(\frac{1}{a} \ln \left(\frac{y}{y_0} \right) + 1 \right), \text{ derived as equation (9)}$$

in chapter five. This variable corresponds to the average earnings the employee expects to earn across the whole wage structure in firm j. The dependance of consumption on this variable is related to Duesenberry's suggestion that individuals tend to emulate their neighbours; in our case they are

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A proper presentation of such a model should include a time parameter t.

greatly influenced by the consumption patterns of the other employees in the firm. This may be a more realistic approach since an employee will associate with his colleagues rather than with his neighbours.

The same variable is also reminiscent of Friedman's permanent income since it represents the expected value of earnings of a homogeneous group subject to a particular income promotion ladder or wage structure.

B. A Final Concluding Note.

In the previous chapters we presented various stochastic models explaining the growth and distribution of labour incomes in segmented labour markets focussing our attention on internal labour markets. Our intention in the present thesis has been neither to provide a curve fitting exercise nor to test empirically a suggested hypothesis related to a specific shape of the distribution of labour incomes.

Although in the literature there are quite a few theories of the size distribution and growth of earnings, no general theory exists that could incorporate observed patterns on the micro-economic level. Neoclassical analysis remains silent at this point since its marginal productivity theory explains the determination of labour incomes - under very stringent conditions - but not their distribution. This thesis has tried to fill this gap by providing the methodological framework within which observed wage structure across firms or income promotion ladders in a firm could be analysed. We do not pretend that we have succeeded in providing an overall theory of the size distribution of labour incomes or in unveiling all the factors that could influence the distribution. Some of these factors are highly qualitative in nature, and to provide a general quantitative theoretical framework seems to be a formidable task that requires further research. Unfortunately the mathematical problems involved and the complicated economic rationale behind any wage structure present serious obstacles which are not easily overcome. We must note, however, that our principal aim has been the derivation of explicit results, where possible, rather than the exploration of, perhaps, more realistic models that would require the application of simulation techniques.

Finally certain parameters of the models such as the rates of entry into the firm and exit, and the income increment have been treated as constants. They could be easily treated as functions of certain economic variables; for instance, the income increment in the basic model of chapter five could be a function of the rate of growth of the firm measured by the volume of sales or by the level of profits. The same principle may be extended and applied to the income promotion structure. The promotion rate (or the hazard function) could be a function of seniority within each income level and of the firm's output, that is $\mu(\tau, q) = \phi(\tau) + \xi(q)$, where $\mu(\tau, q)$ is the promotion rate, $\phi(\tau)$ a function related to seniority within each income level and $\xi(q)$ is a function of output. Then we can apply the procedure followed in our basic model without the mathematical results being affected.

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